

1.3-GHz Superconducting Radio Frequency Cavity Stress and Buckling Analysis

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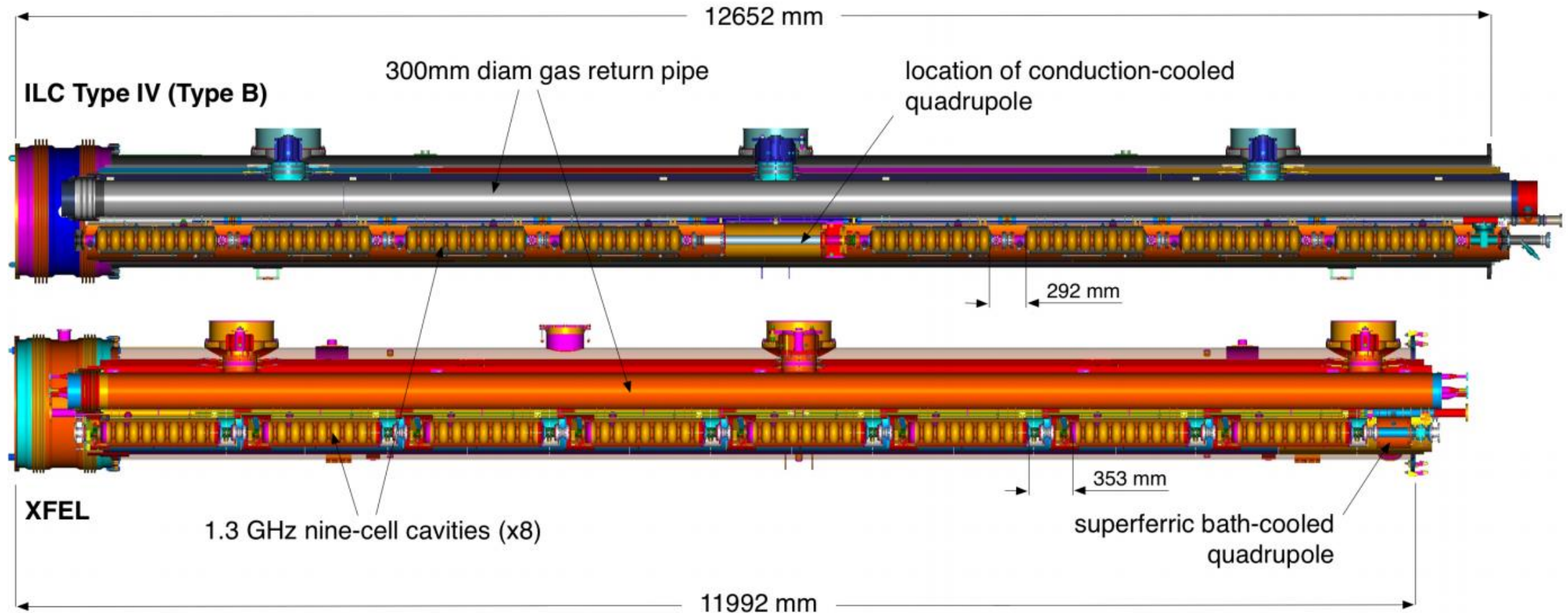
Contents



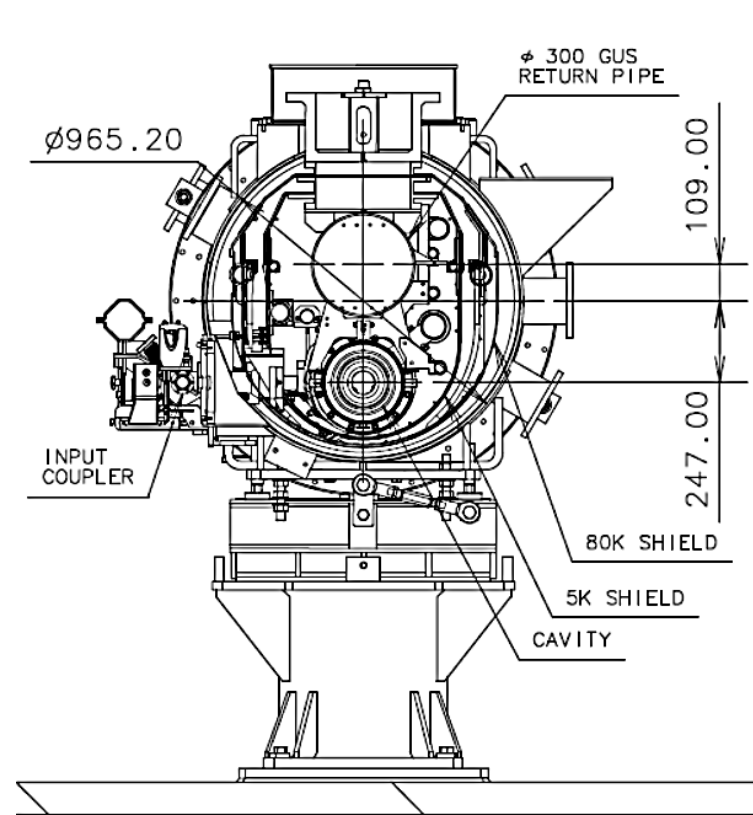
1. Cryomodule Assembly
2. Pressure Vessel Compliance for Cryomodules
3. Structural Analysis using FEA
4. 1.3 GHz 3-Cell SRF Cavity Structural Analysis
5. Eigen value Buckling Analysis
6. 1.3 GHz 3-Cell SRF Cavity Buckling Analysis
7. 1.3 GHz 3-Cell SRF Cavity Non-linear Buckling Analysis

Cryomodule Assembly

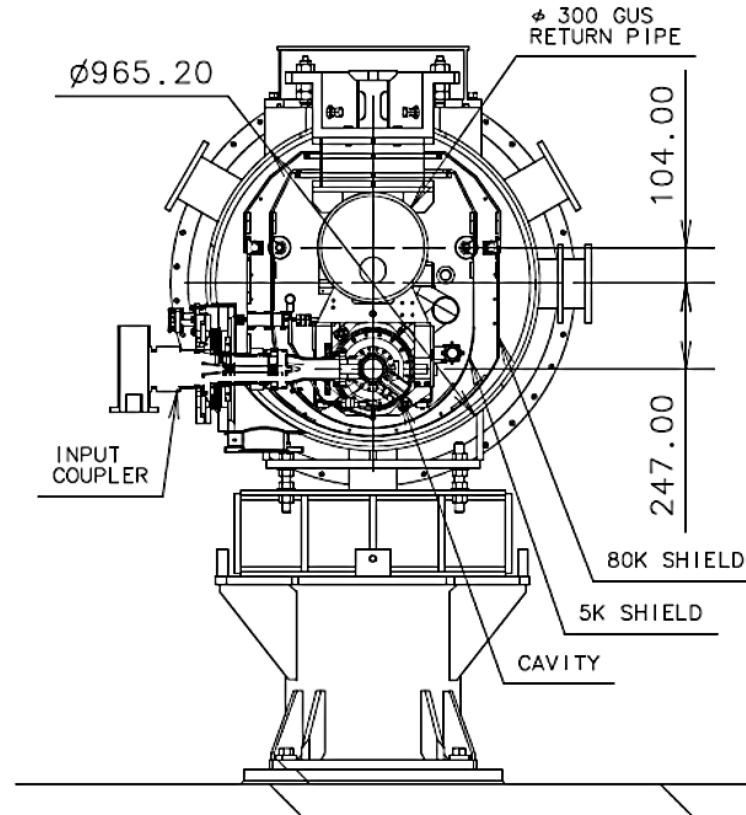
ILC Cryomodule



Cryomodule Cross-sectional View



(a) Module-C for FNAL cavity



(b) Module-A for KEK cavity

- 2 K two-phase superfluid helium pipeline
- 2 K gas helium return line.
- 5 K shield line and 80 K or 40 K shield line.
- Cold mass at 2 K is enclosed with two thermal shields
- No issues with pressure vessel compliance for these lines.

Pressure Vessel Compliance

1.3 GHz 9-Cell SRF Cavity Assembly

Niobium SRF Cavity with its Titanium jacket is considered as a pressure vessel



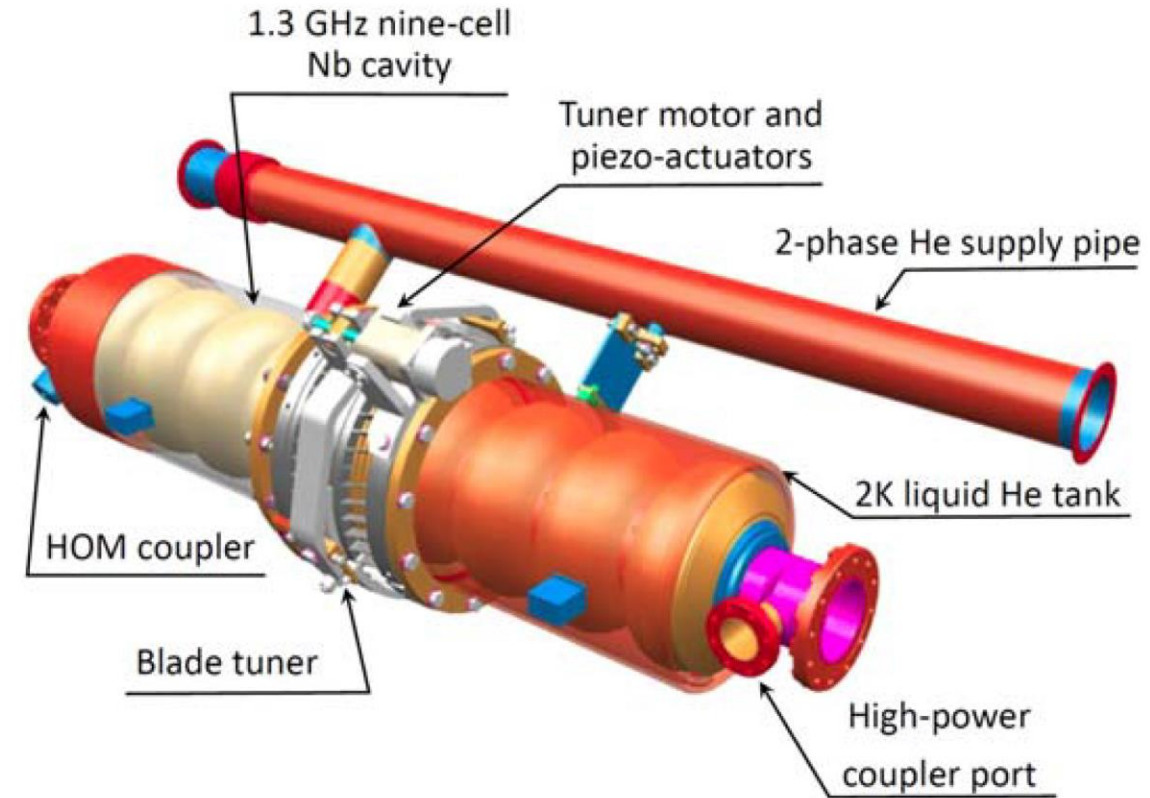
Design must be cleared by high-pressure gas safety authorities



Determine maximum allowable stress and buckling pressure using ANSYS.



Nb and NbTi not listed as a material for high pressure vessel design



Cited from: The International Linear Collider: A Global Project, arXiv:1903.01629 [hep-ex]

Steps Involved in Preparing Documentation



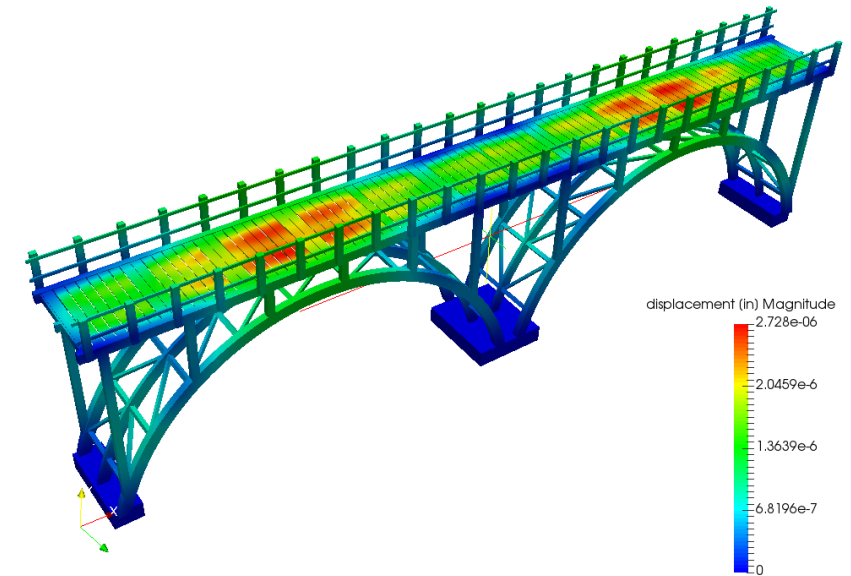
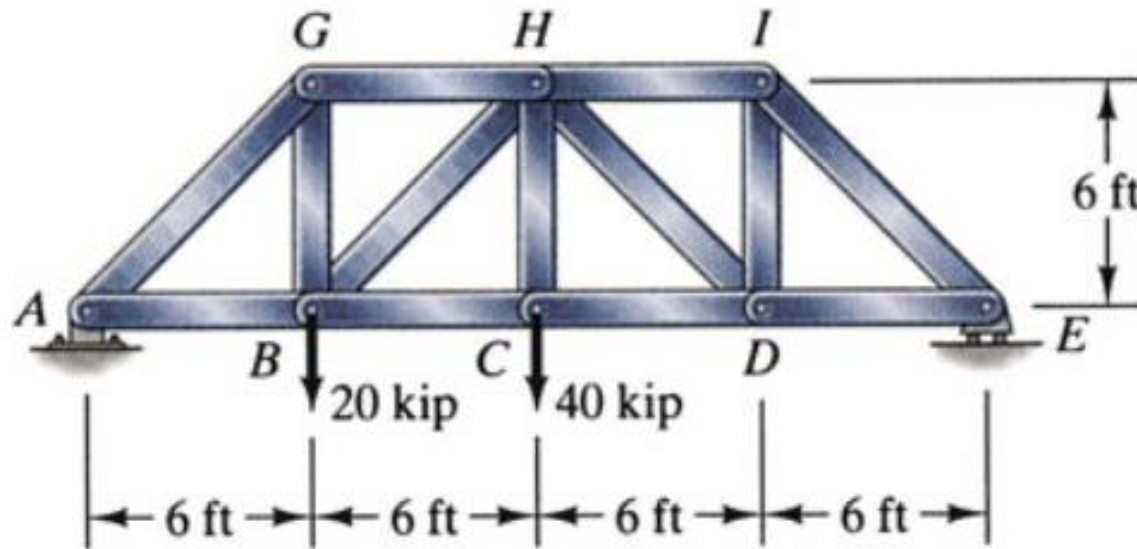
- Description of the SRF cavity assembly.
- Mechanical properties of the materials at room and in liquid helium temperatures.
- Mechanical properties of welded joints like Nb-Nb, Nb-Ti, Ti-Ti welds etc.
- Stress and buckling analysis of the cavity assembly using CAE software at maximum allowable working pressure (0.2 MPa) and tuner displacement.
- Cavity fabrication information.
- Pressure test and examination reports.
- Documentation to summarize above items to be submitted to the high-pressure gas safety authority.

Structural Analysis

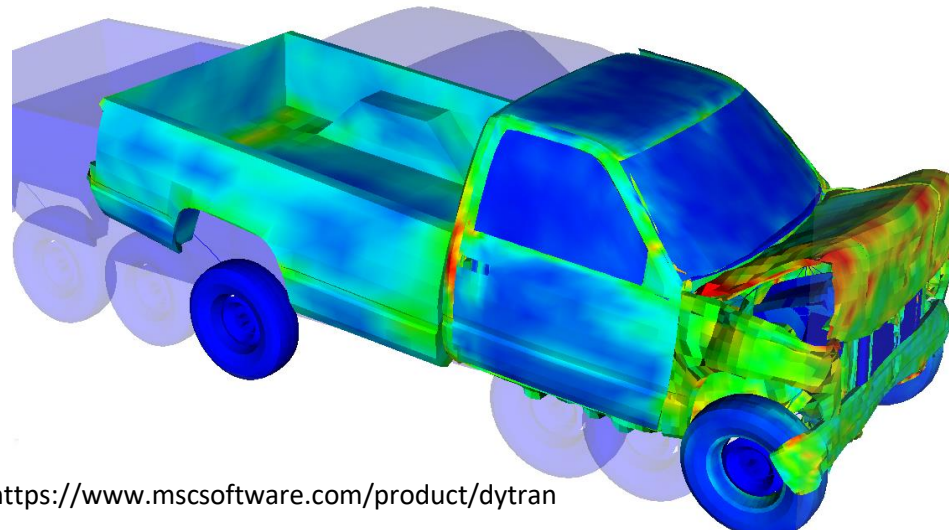
Structural Analysis



- Structural analysis determines the integrity of structure to withstand loads.
- Structural analysis incorporates the fields of mechanics and dynamics as well as the many failure theories.
- From a theoretical perspective, the primary goal of structural analysis is the computation of deformations, internal forces, and stresses.
- In practice, structural analysis reveals the structural performance of the engineering design and ensures the soundness of structural integrity in design without dependence on direct testing.



A. Bedford and K.M Leichti, Mechanics of materials, second edition, Springer.



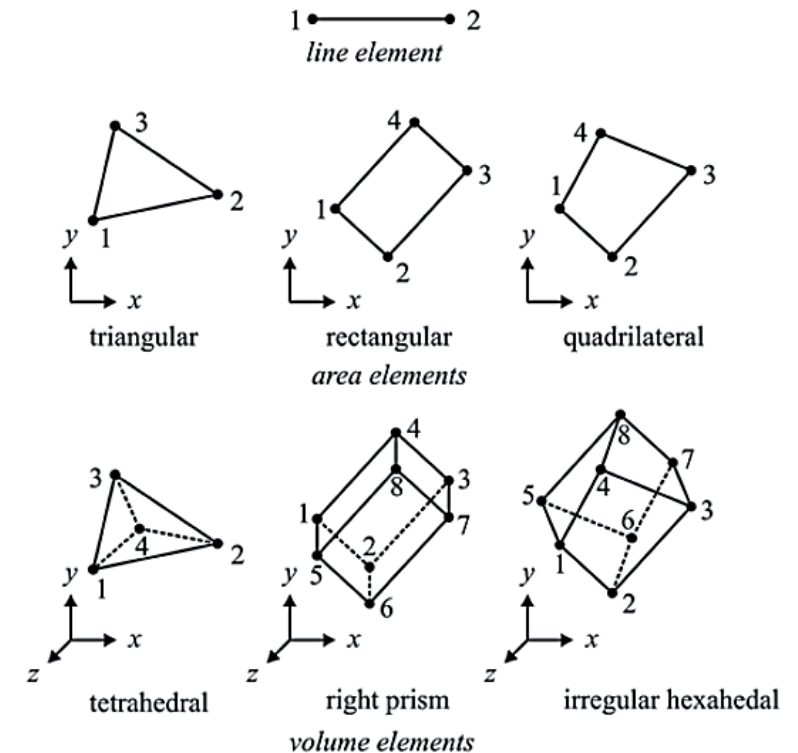
Finite Element Analysis



- FEA is a powerful computational technique for approximate solutions to a variety of “real-world” problems.
- It relies on decomposition of domain (solid, liquid or gas) into a finite number of sub-domains.
- Reduces the problem into finite number of unknowns by dividing the domain into elements and by expressing the unknown field variable in terms of the assumed approximating functions within each element.

Finite Element Analysis

- Discretization of the domain into finite number of sub-domains.
- Selection of interpolating functions.
- Development of the elemental matrix of the sub-domain.
- Assembly of the element matrices for each subdomain to obtain global matrix.
- Imposition of boundary conditions.
- Solution of equations.
- Additional data-analysis if necessary.



Cited from: E Madenci and I Guven., The finite element method and applications in engineering using ANSYS®, second edition, Springer 2015.

Direct Approach: Linear Spring System

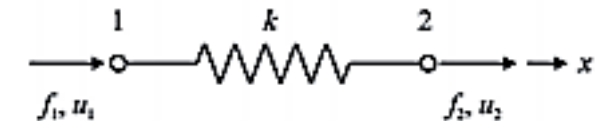
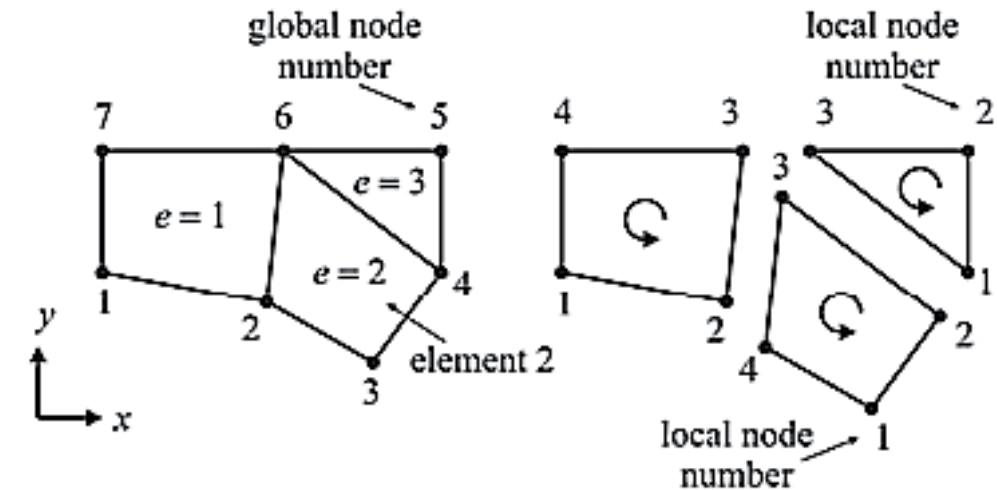
- Suitable for simpler problems but fundamental step of a typical FEA.
- Global system of equations can be cast into:

$$Ku = F$$

$$u = u_1 - u_2 \quad \rightarrow \quad f_1 = ku = k(u_1 - u_2) \quad \rightarrow \quad f_2 = -f_1$$

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \text{ or } k^{(e)} u^{(e)} = f^{(e)}$$

Here, $u^{(e)}$ is the vector of nodal unknowns representing displacement, $k^{(e)}$ is the element (stiffness) matrix and $f^{(e)}$ is the element (force) vector. The stiffness matrix can be represented in its indicial form as $k_{ij}^{(e)}$.



Cited from: E Madenci and I Guven., The finite element method and applications in engineering using ANSYS®, second edition, Springer 2015.

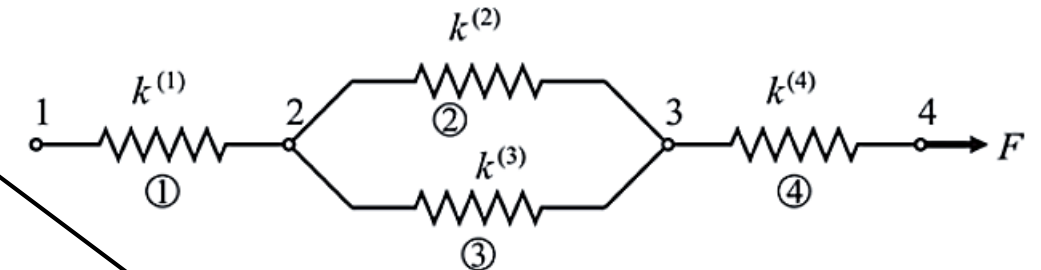
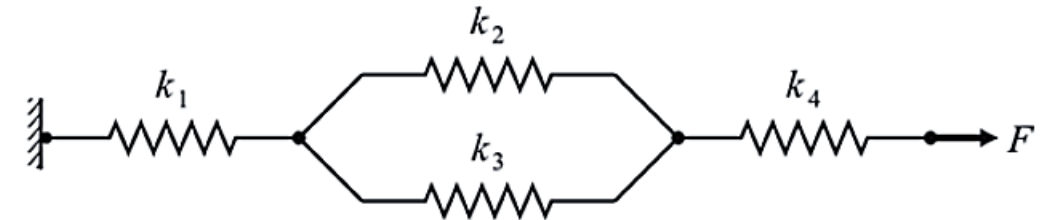
Assembly of Global System Matrix

- Engineering problem model with finite elements requires the assembly of element characteristic (stiffness) matrices and element right-hand-side vectors.

$$K = \sum_{e=1}^E k^{(e)}$$

$$Ku = F$$

$$F = \sum_{e=1}^E f^{(e)}$$



$$K = \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} & 0 & 0 \\ k_{21}^{(e)} & (k_{22}^{(1)} + k_{11}^{(2)} + k_{11}^{(3)}) & (k_{12}^{(2)} + k_{12}^{(3)}) & 0 \\ 0 & (k_{21}^{(2)} + k_{21}^{(3)}) & (k_{22}^{(2)} + k_{22}^{(3)} + k_{11}^{(4)}) & k_{12}^{(e)} \\ 0 & 0 & k_{21}^{(e)} & k_{22}^{(4)} \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} u_1^{(1)} \\ u_2^{(1)} = u_1^{(2)} = u_1^{(3)} \\ u_2^{(2)} = u_2^{(3)} = u_1^{(4)} \\ u_2^{(4)} \end{bmatrix}$$

$$F = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} f_1^{(1)} \\ f_2^{(1)} + f_1^{(2)} + f_1^{(3)} \\ f_2^{(2)} + f_2^{(3)} + f_1^{(4)} \\ f_2^{(4)} \end{bmatrix}$$

Cited from: E Madenci and I Guven., The finite element method and applications in engineering using ANSYS®, second edition, Springer 2015.

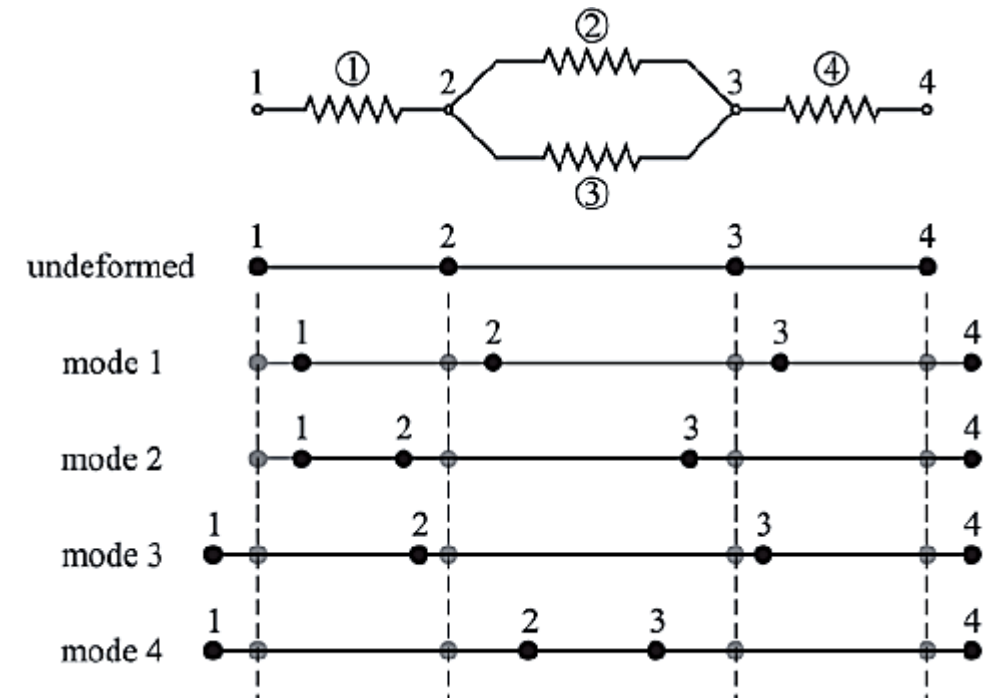
Solution of Global Matrix

- For the specific values $k_{11}^{(e)} = k_{22}^{(e)} = k^{(e)}$ and $k_{21}^{(e)} = k_{12}^{(e)} = -k^{(e)}$, the global system matrix becomes

$$K = k^{(e)} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

- The eigenvalues are $\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 3 - \sqrt{5}$, and $\lambda_4 = 3 + \sqrt{5}$. The corresponding eigenvectors are

$$u^{(1)} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, u^{(2)} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, u^{(3)} = \begin{bmatrix} 1 \\ 2 - \sqrt{5} \\ -2 + \sqrt{5} \\ 1 \end{bmatrix}, u^{(4)} = \begin{bmatrix} 1 \\ 2 + \sqrt{5} \\ -2 - \sqrt{5} \\ 1 \end{bmatrix}$$



Cited from: E Madenci and I Guven., The finite element method and applications in engineering using ANSYS®, second edition, Springer 2015.

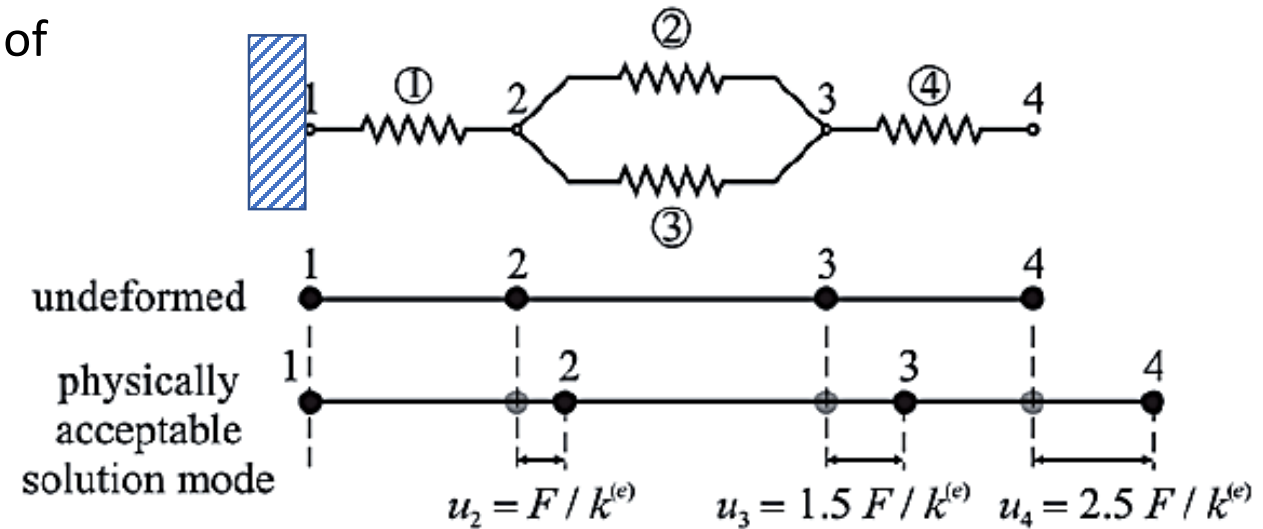
Applying Boundary Condition

- Possible to obtain unique solution with boundary conditions.
- If $u_1 = 0$, the nodal force f_1 still remains an unknown and the corresponding nodal forces have values of $f_2 = f_3 = 0$, and $f_4 = F$.

$$k^{(e)} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} f_1 \\ 0 \\ 0 \\ F \end{bmatrix}$$



$$u_2 = \frac{F}{k^{(e)}}, u_3 = \frac{3}{2} \frac{F}{k^{(e)}}, u_4 = \frac{5}{2} \frac{F}{k^{(e)}}$$



Cited from: E Madenci and I Guven., The finite element method and applications in engineering using ANSYS®, second edition, Springer 2015.

Stress-strain Relationship in ANSYS

- Stress-strain relationship in ANSYS for linear materials is defined as:

$$\{\sigma\} = D\{\varepsilon^{el}\}$$

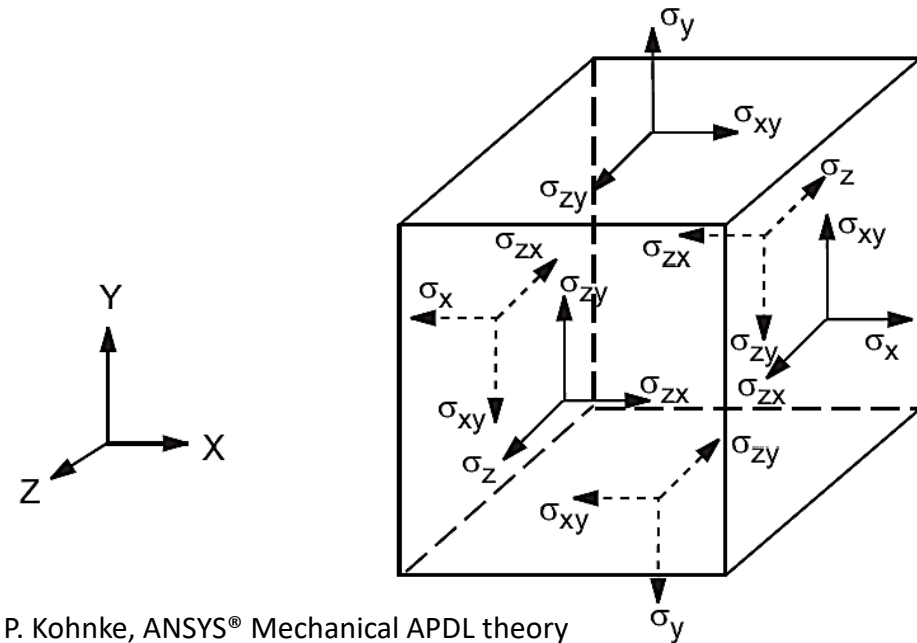
$\{\sigma\}$ = Stress vector = $[\sigma_x \sigma_y \sigma_z \sigma_{xy} \sigma_{yz} \sigma_{xz}]^T$

$[D]$ = Elasticity or elastic stiffness matrix or stress-strain matrix.

$\{\varepsilon^{el}\} = \{\varepsilon\} - \{\varepsilon^{th}\}$ = elastic strain vector

$\{\varepsilon\}$ = total strain vector = $[\varepsilon_x \varepsilon_y \varepsilon_z \varepsilon_{xy} \varepsilon_{yz} \varepsilon_{xz}]^T$

$\{\varepsilon^{th}\}$ = thermal strain vector



Cited from: P. Kohnke, ANSYS® Mechanical APDL theory reference, ANSYS® Inc.

- Also,

$$\{\varepsilon\} = \{\varepsilon^{th}\} + [D]^{-1}\{\sigma\},$$

- For the 3-D case, the thermal strain vector is:

$$\{\varepsilon^{th}\} = \Delta T [\alpha_x^{se} \alpha_y^{se} \alpha_z^{se} 0 0 0]^T,$$



α_x^{se} = Secant coefficient of thermal expansion in the x direction

$$\Delta T = T - T_{ref}$$

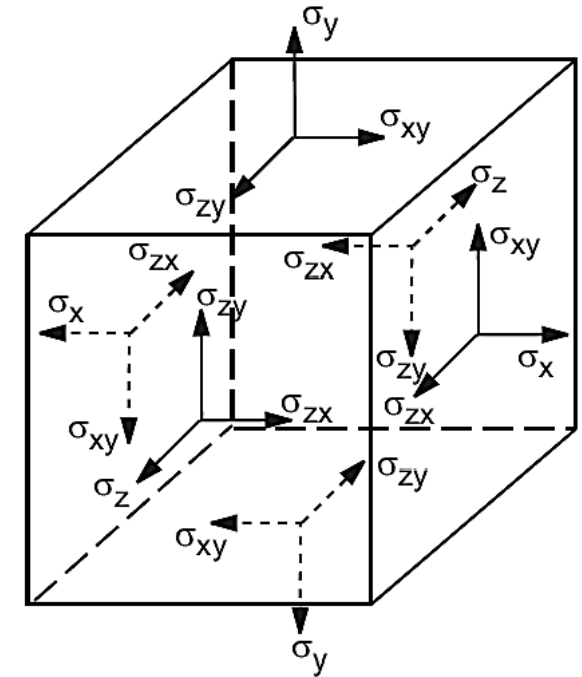
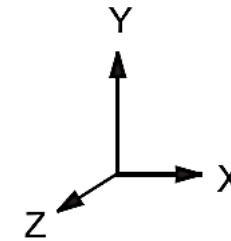
T = current temperature at the point in question

T_{ref} = reference strain free temperature

Stress-strain Relationship in ANSYS

- The flexibility or compliance matrix for the linear material is:

$$[D]^{-1} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & -\frac{\nu_{xz}}{E_x} & 0 & 0 & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & -\frac{\nu_{yz}}{E_y} & 0 & 0 & 0 \\ \frac{1}{E_z} & -\frac{\nu_{zx}}{E_z} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} \end{bmatrix}$$



Where the typical terms are:

E_x = Young's modulus in the x direction, ν_{xy} = major Poisson's ratio

ν_{yx} = minor Poisson's ratio, G_{xy} = Shear modulus in the xy plane

For isotropic materials ($E_x = E_y = E_z$ and $\nu_{xy} = \nu_{yz} = \nu_{xz}$)

Cited from: P. Kohnke, ANSYS® Mechanical APDL theory reference, ANSYS® Inc.

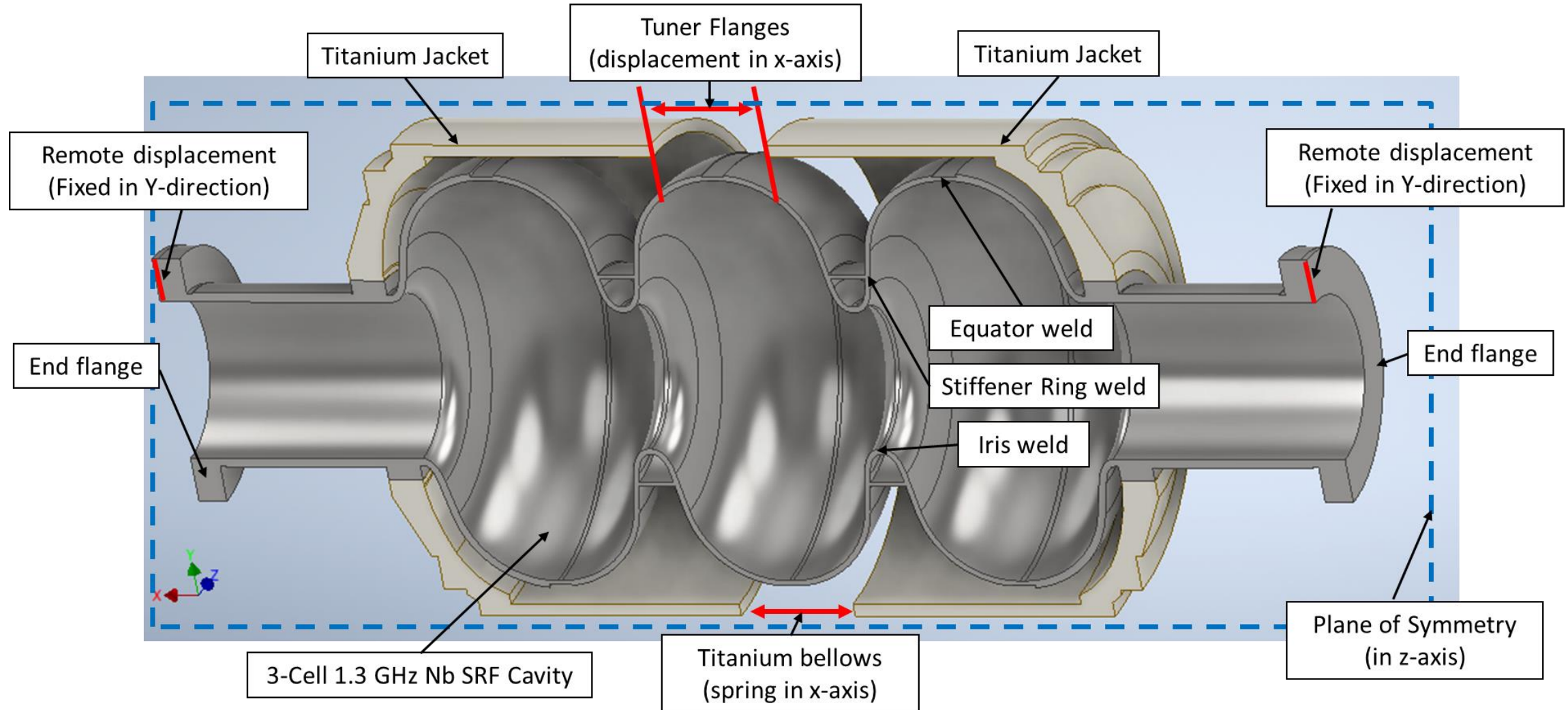
1.3-GHz 3-Cell SRF Cavity Stress Analysis

Note



- The results presented here are just an example and the results presented here are not supposed to be considered for 9-Cell SRF Cavity structure.

SRF Cavity Assembly

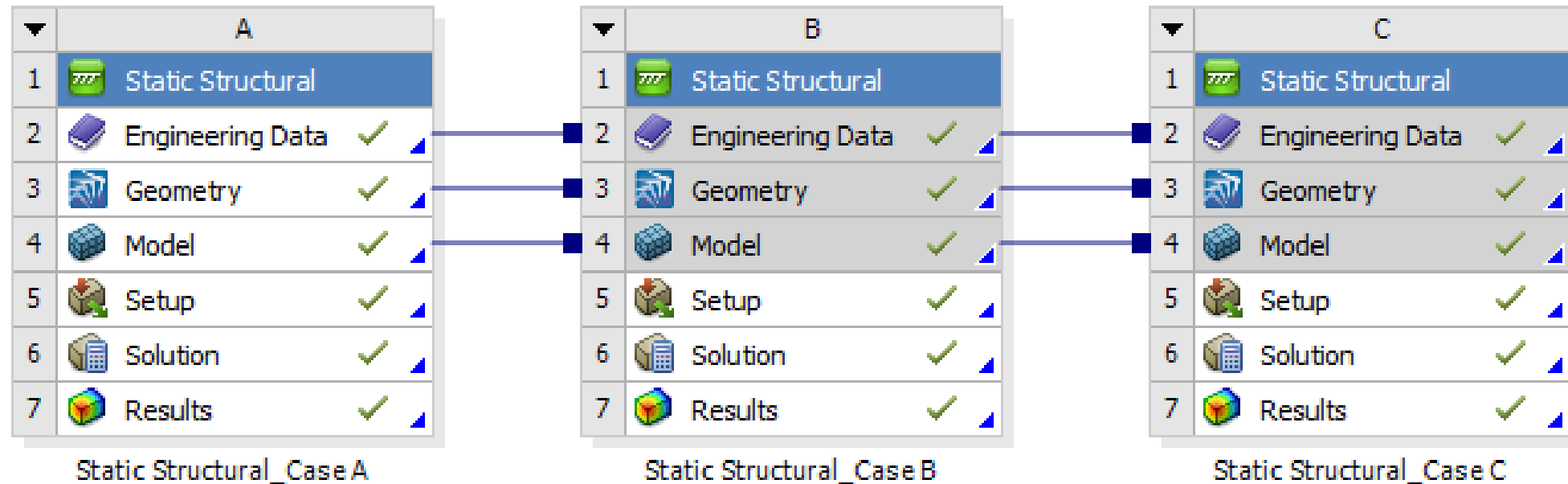


Mechanical Properties

Properties	Unit	Niobium	Titanium
Young's Modulus	GPa	103	107
Poisson's Ratio	-	0.38	0.32
Coefficient of Thermal Expansion	$\times 10^{-6} / ^\circ\text{C}$	4.87	5.22

- Elastic properties of the materials will be considered for structure and buckling analysis.
- Coefficient of thermal expansion necessary to include contraction of material during cooldown.

Case Structure in ANSYS



Case Study	Pressure	Tuner Displacement	Temperature
Case A	0.2 MPa	0.65 mm	40 °C
Case B	0.2 MPa	0.65 mm	-271.4 °C
Case C	0.2 MPa	3 mm	-271.4 °C

Engineering Data

Niobium Properties

Outline of Schematic A2, B2, C2: Engineering Data					
	A	B	C	D	E
1	Contents of Engineering Data			Source	Description
2	Material				
3	NIOBIUM				"Equation of State and Strength Properties of Selected Materials". Steinberg D.J. LLNL. Feb 1991
4	TITANIUM				"Equation of State and Strength Properties of Selected Materials". Steinberg D.J. LLNL. Feb 1991
*	Click here to add a new material				

Properties of Outline Row 3: NIOBIUM					
	A	B	C	D	E
1	Property	Value	Unit		
2	Material Field Variables	Table			
3	Density	8590	kg m ⁻³		
4	Isotropic Secant Coefficient of Thermal Expansion				
5	Coefficient of Thermal Expansion	4.87E-06	C ⁻¹		
6	Isotropic Elasticity				
7	Derive from	Young's ...			
8	Young's Modulus	1.03E+05	MPa		
9	Poisson's Ratio	0.38			
10	Bulk Modulus	1.4306E+11	Pa		
11	Shear Modulus	3.7319E+10	Pa		

Titanium Properties

Outline of Schematic A2, B2, C2: Engineering Data						
	A	B	C	D	E	
1	Contents of Engineering Data				Source	Description
2	Material					
3	NIOBIUM				E	"Equation of State and Strength Properties of Selected Materials". Steinberg D.J. LLNL. Feb 1991
4	TITANIUM				E	"Equation of State and Strength Properties of Selected Materials". Steinberg D.J. LLNL. Feb 1991
*	Click here to add a new material					

Properties of Outline Row 4: TITANIUM					
	A	B	C	D	E
1	Property	Value	Unit		
2	Material Field Variables	Table			
3	Density	4510	kg m ⁻³		
4	Isotropic Secant Coefficient of Thermal Expansion				
5	Coefficient of Thermal Expansion	5.22E-06	C ⁻¹		
6	Isotropic Elasticity				
7	Derive from	Young's ...			
8	Young's Modulus	1.07E+05	MPa		
9	Poisson's Ratio	0.32			
10	Bulk Modulus	9.9074E+10	Pa		
11	Shear Modulus	4.053E+10	Pa		

Geometry

Structure

- SYS*
- Nb SRF Cavity
- Ti tank_1
- Ti tank_2

Structure | Layers | Selection | Groups | Views

Options - Selection

Properties

- Analysis
 - Share Topology: Share
- Document
 - Display Name: SYS
 - Document Path: C:\Users\ashish\Desktop\Conferences
 - Locked: False
 - Use File Name: True
- File
 - Category
 - Content Status
 - Content Type
 - Created: 7/20/2021 05:10 PM
 - Creator
 - Description
 - Identifier
 - Keywords
 - Language
 - Last Modified By
 - Last Printed
 - Modified: 7/20/2021 05:10 PM
 - Revision
 - Subject
 - Title
 - Version
- Material
 - Material Name: Unknown Material
 - Fluid: False
 - Density: None
- Sheet Metal
 - Sheet Metal: False

Properties | Appearance

1 Static Structural

2 Engineering Data

3 **Geometry**

4 Model

5 Setup

6 Solution

7 Results

Click an object. Double-click to select an edge loop. Triple-click to select a solid.

Structure

- SYS*
- Nb SRF Cavity
- Ti tank_1
- Ti tank_2

Structure | Layers | Selection | Groups | Views

Options - Selection

Properties

- Analysis
 - Share Topology: Share
- Document
 - Display Name: SYS
 - Document Path: C:\Users\ashish\Desktop\Conferences
 - Locked: False
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 - Last Modified By
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 - Modified: 7/20/2021 05:10 PM
 - Revision
 - Subject
 - Title
 - Version
- Material
 - Material Name: Unknown Material
 - Fluid: False
 - Density: None
- Sheet Metal
 - Sheet Metal: False

Properties | Appearance

- CAD Model is uploaded to Geometry
- Topology is set to Share

Model – Material Assignment

Project Schematic (Left):

- Model (A4, B4, C4)
 - Geometry
 - SYS
 - Nb SRF Cavity
 - Ti tank_1
 - Ti tank_2

Details of "Nb SRF Cavity" (Right):

Definition	
Suppressed	No
Stiffness Behavior	Flexible
Coordinate System	Default Coordinate System
Reference Temperature	By Environment
Behavior	None

Material	
Assignment	NIOBIUM
Nonlinear Effects	Yes
Thermal Strain Effects	Yes

Messages (Bottom Right):

- Warning: Not enough constraints appear to be applied to prevent rigid body
- Warning: Large deformation effects are active which may have invalidated
- Warning: Not enough constraints appear to be applied to prevent rigid body
- Warning: Large deformation effects are active which may have invalidated
- Warning: Not enough constraints appear to be applied to prevent rigid body
- Warning: Large deformation effects are active which may have invalidated

- Material is assigned to each geometry.
- Stiffness behavior is set to Flexible.

Model – Symmetry

	A
1	Static Structural
2	Engineering Data
3	Geometry
4	Model
5	Setup
6	Solution
7	Results

Outline

Filter: Name

- Project
 - Model (A4, B4, C4)
 - Geometry
 - Coordinate Systems
 - Symmetry
 - Symmetry Region
 - Connections
 - Mesh
 - Named Selections
 - Static Structural (A5)
 - Analysis Settings
 - Pressure
 - Displacement
 - Displacement 2
 - Remote Displacement
 - Remote Displacement 2
 - Standard Earth Gravity
 - Solution (A6)
 - Solution Information
 - Equivalent Stress
 - Stress Intensity
 - Equivalent Elastic Strain
 - Elastic Strain Intensity
 - Total Deformation
 - Static Structural 2 (B5)
 - Analysis Settings

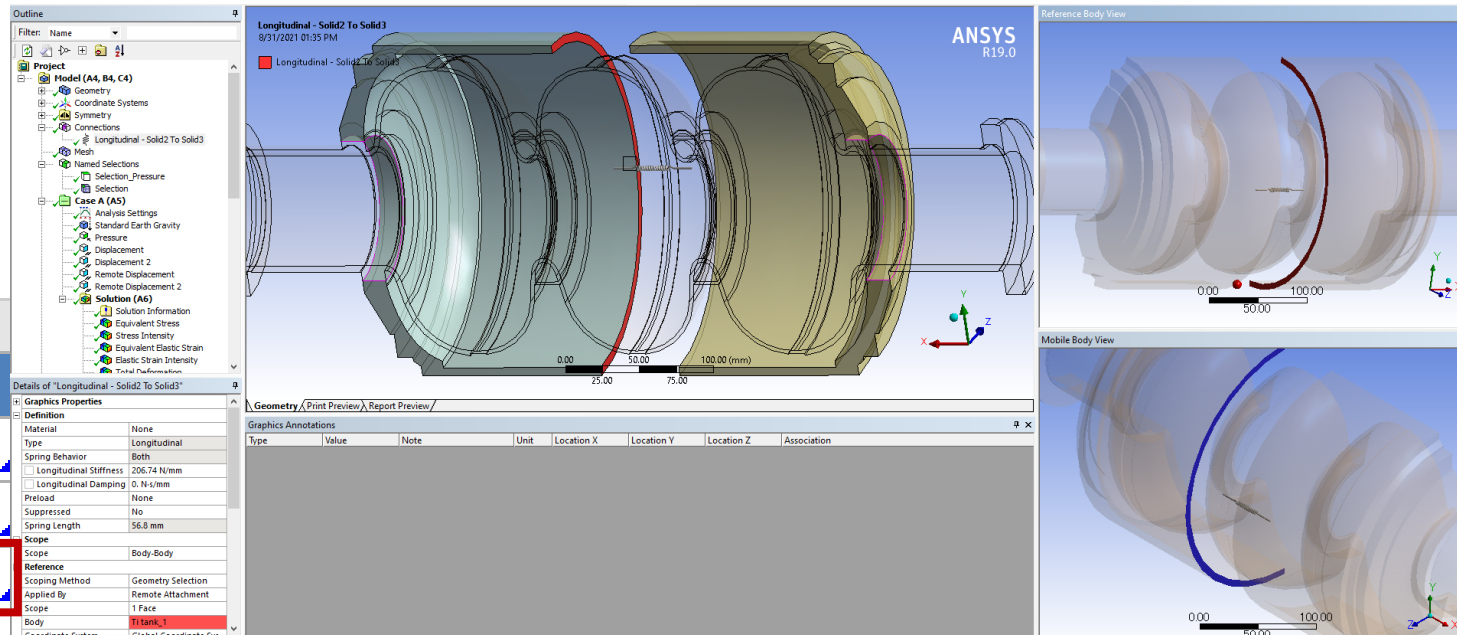
Details of "Symmetry Region"

Scope	
Scoping Method	Geometry Selection
Geometry	6 Faces
Definition	
Scope Mode	Manual
Type	Symmetric
Coordinate System	Global Coordinate System
Symmetry Normal	Z Axis
Suppressed	No

- Model symmetry along z-axis defined.
- Faces in Red specifies the symmetry region.

Model – Virtual Spring

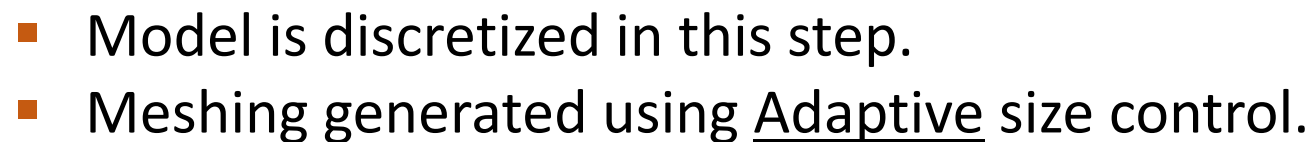
	A
1	Static Structural
2	Engineering Data
3	Geometry
4	Model
5	Setup
6	Solution
7	Results



- A virtual spring replaces Ti bellow between Ti jacket with stiffness of 206.7 N/mm.

Details of "Longitudinal - Solid2 To Solid3"	
Graphics Properties	
Definition	
Material	None
Type	Longitudinal
Spring Behavior	Both
☐ Longitudinal Stiffness	206.74 N/mm
☐ Longitudinal Damping	0. N-s/mm
Preload	None
Suppressed	No
Spring Length	56.8 mm
Scope	
Scope	Body-Body
Reference	
Scoping Method	Geometry Selection
Applied By	Remote Attachment
Scope	1 Face
Body	Ti tank_1
Coordinate System	Global Coordinate System
Reference X Coordinate	3073.1 mm
Reference Y Coordinate	2835. mm
Reference Z Coordinate	74.563 mm
Reference Location	Click to Change
Behavior	Deformable
Pinball Region	All
Mobile	
Scoping Method	Geometry Selection
Applied By	Remote Attachment
Scope	1 Face
Body	Ti tank_2
Coordinate System	Global Coordinate System
Mobile X Coordinate	3016.3 mm
Mobile Y Coordinate	2835. mm
Mobile Z Coordinate	74.563 mm
Mobile Location	Click to Change
Behavior	Deformable
Pinball Region	All

Center for
Applied
Superconducting
Accelerator
応用超伝導加速器センター



Setup – Analysis Settings

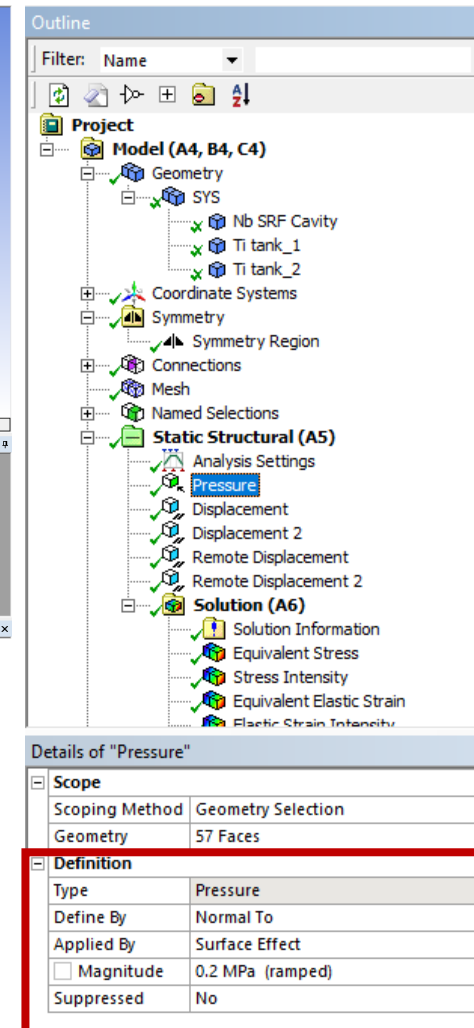
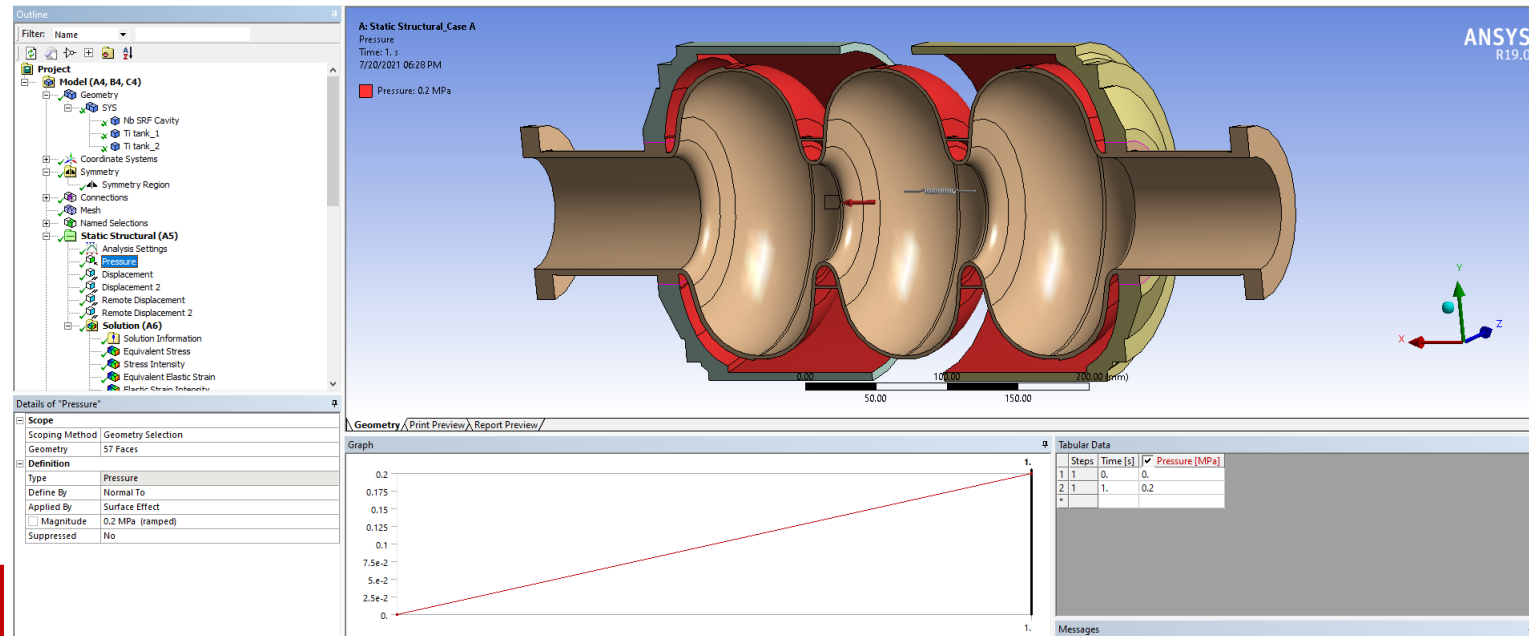
	A
1	Static Structural
2	Engineering Data
3	Geometry
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Details of "Analysis Settings"

Step Controls	
Number Of Steps	1.
Current Step Number	1.
Step End Time	1. s
Auto Time Stepping	Program Controlled
Solver Controls	
Solver Type	Program Controlled
Weak Springs	Off
Solver Pivot Checking	Program Controlled
Large Deflection	On
Inertia Relief	Off
Rotordynamics Controls	
Restart Controls	
Nonlinear Controls	
Output Controls	
Analysis Data Management	
Visibility	

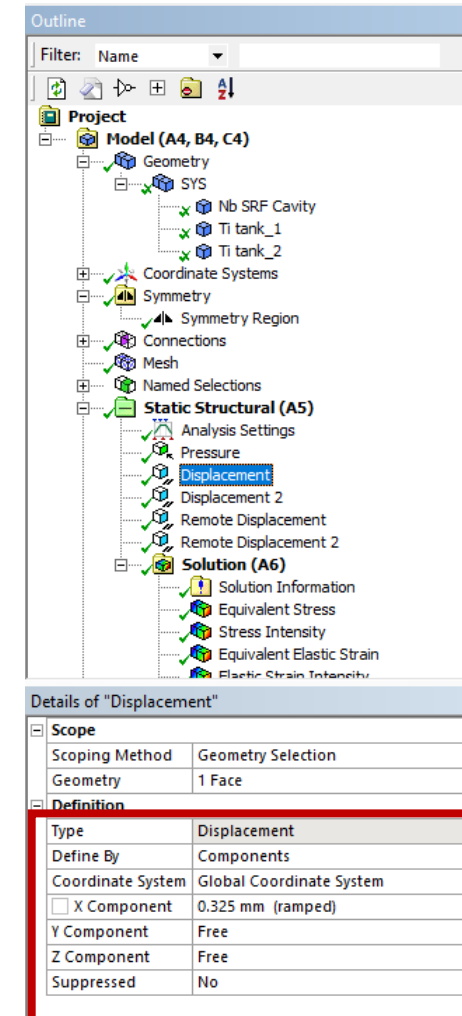
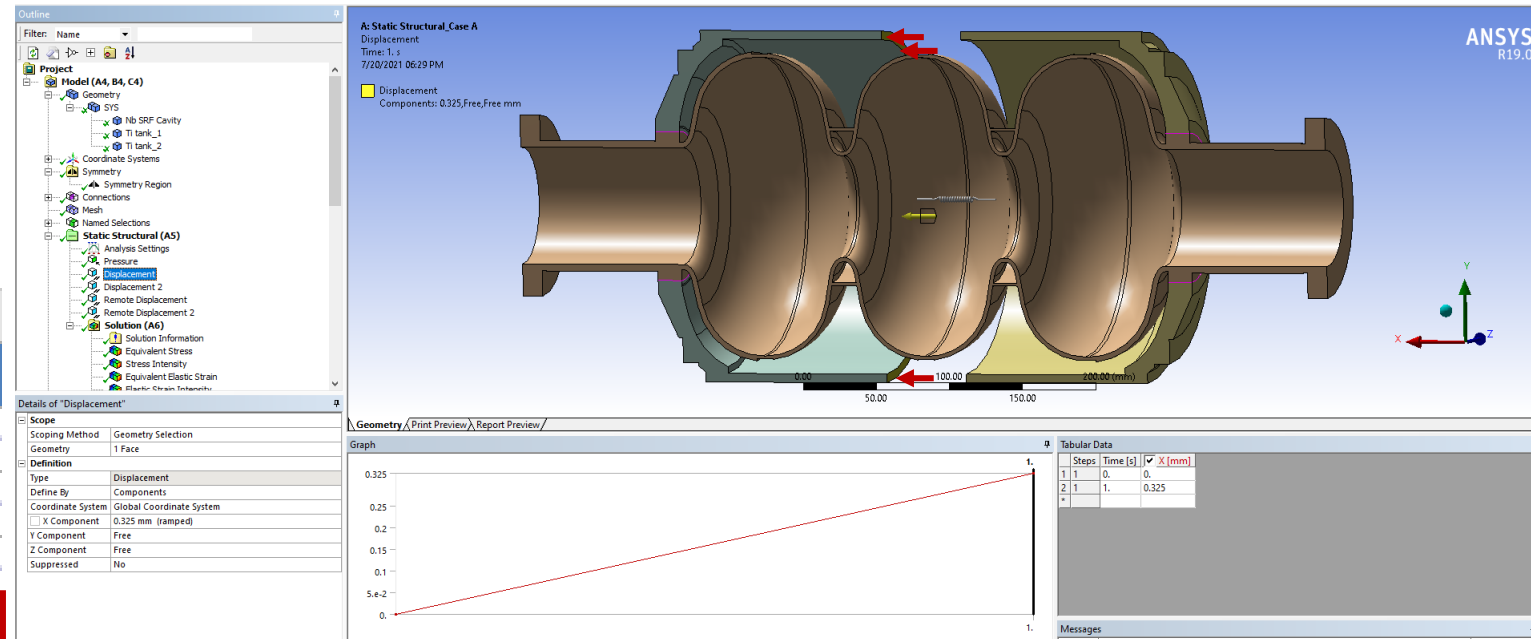
- Set number of steps, sub-steps etc.
- Set Large deflection to ON, when non-linear analysis is expected.

Setup – Pressure Boundary Condition



- Pressure boundary condition (BC) to simulate pressurized helium between SRF cavity and Ti jacket at 0.2 MPa.
- Surfaces in RED are provided with pressure boundary condition.

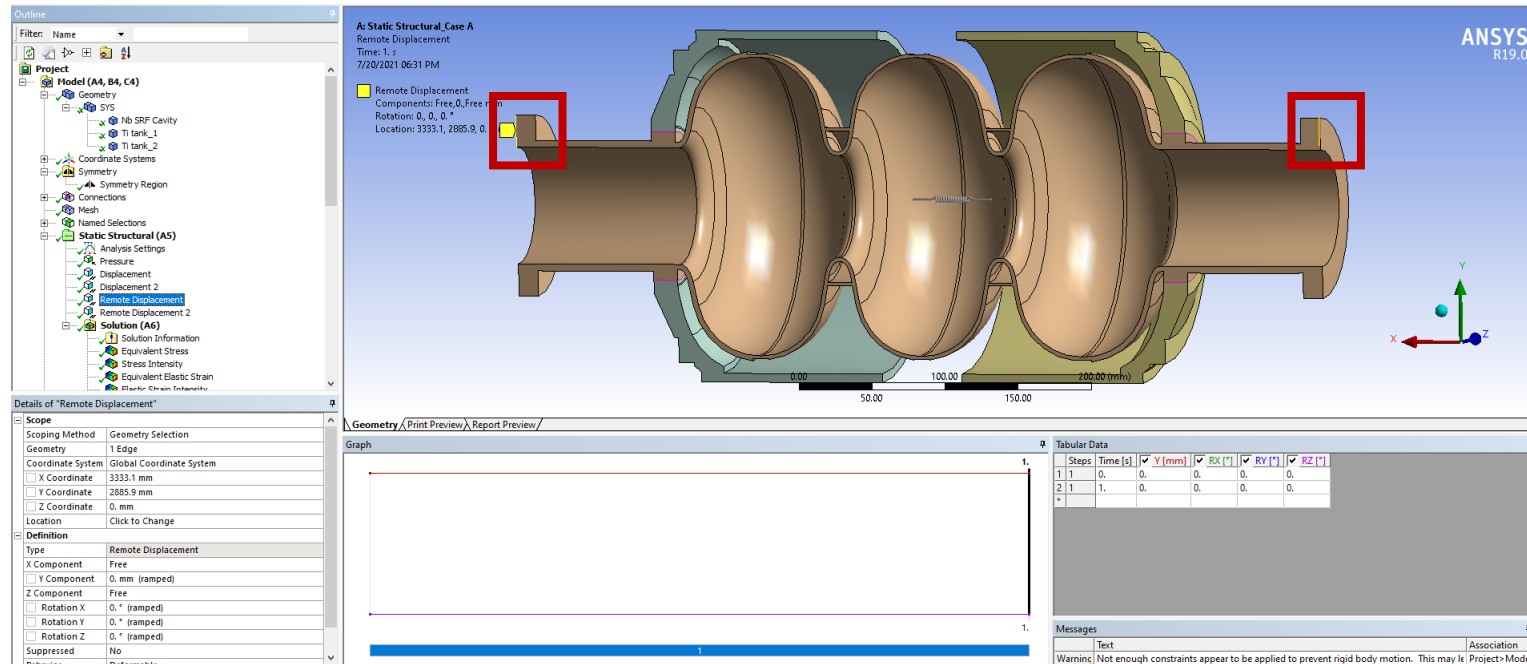
Setup – Displacement Boundary Condition



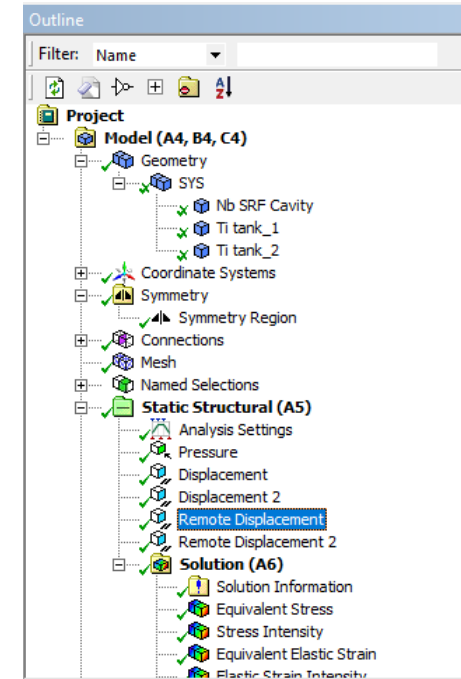
- Displacement BC to simulate tuner movement.
- Displacement in X-direction and movement in other axis are not constrained.

Setup – Remote Displacement Boundary Condition

	A
1	Static Structural
2	Engineering Data
3	Geometry
4	Model
5	Setup
6	Solution
7	Results



- Remote Displacement BC is similar to displacement but with more control on DOF.
- Line contact on end flanges movement is fixed in Y-axis and no rotation is allowed along any direction.



Details of "Remote Displacement"	
Scoping Method	Geometry Selection
Geometry	1 Edge
Coordinate System	Global Coordinate System
☐ X Coordinate	3333.1 mm
☐ Y Coordinate	2885.9 mm
☐ Z Coordinate	0. mm
Location	Click to Change

Definition	
Type	Remote Displacement
X Component	Free
☐ Y Component	0. mm (ramped)
Z Component	Free
☐ Rotation X	0. ° (ramped)
☐ Rotation Y	0. ° (ramped)
☐ Rotation Z	0. ° (ramped)
Suppressed	No

Setup – Temperature Boundary Condition

	A
1	Static Structural
2	Engineering Data
3	Geometry
4	Model
5	Setup
6	Solution
7	Results

B: Static Structural_Case B
Thermal Condition
Time: 1. s
9/7/2021 09:03 AM
Thermal Condition: -271.4 °C

Details of "Thermal Condition"

Scope	
Scoping Method	Geometry Selection
Geometry	3 Bodies

Definition	
Type	Thermal Condition
Magnitude	-271.4 °C (ramped)
Suppressed	No

Graph

Tabular Data

Steps	Time [s]	Temperature [°C]
1	0.	22.
2	1.	-271.4

Outline

Filter: Name

- Case B (B5)
 - Analysis Settings
 - Standard Earth Gravity
 - Pressure
 - Displacement
 - Displacement 2
 - Remote Displacement
 - Remote Displacement 2
 - Thermal Condition
- Solution (B6)
 - Solution Information
 - Equivalent Stress
 - Stress Intensity
 - Equivalent Elastic Strain
 - Elastic Strain Intensity
 - Total Deformation
- Static Structural 3 (C5)
 - Analysis Settings
 - Pressure
 - Displacement
 - Displacement 2
 - Remote Displacement
 - Remote Displacement 2
 - Thermal Condition
- Solution (C6)
 - Solution Information
 - Equivalent Stress
 - Stress Intensity
 - Equivalent Elastic Strain

Details of "Thermal Condition"

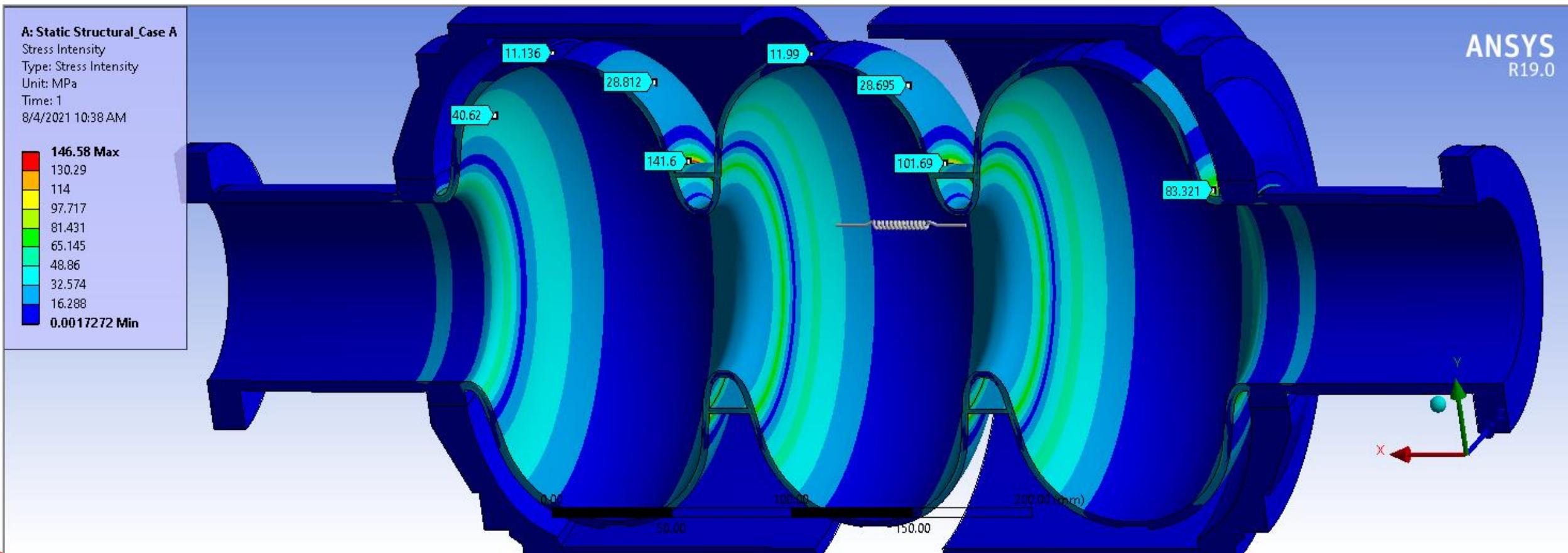
Scope	
Scoping Method	Geometry Selection
Geometry	3 Bodies

Definition	
Type	Thermal Condition
Magnitude	-271.4 °C (ramped)
Suppressed	No

- Applied for Case B and C to simulate cooldown from room temperature to -271.4 °C.
- All bodies are selected.

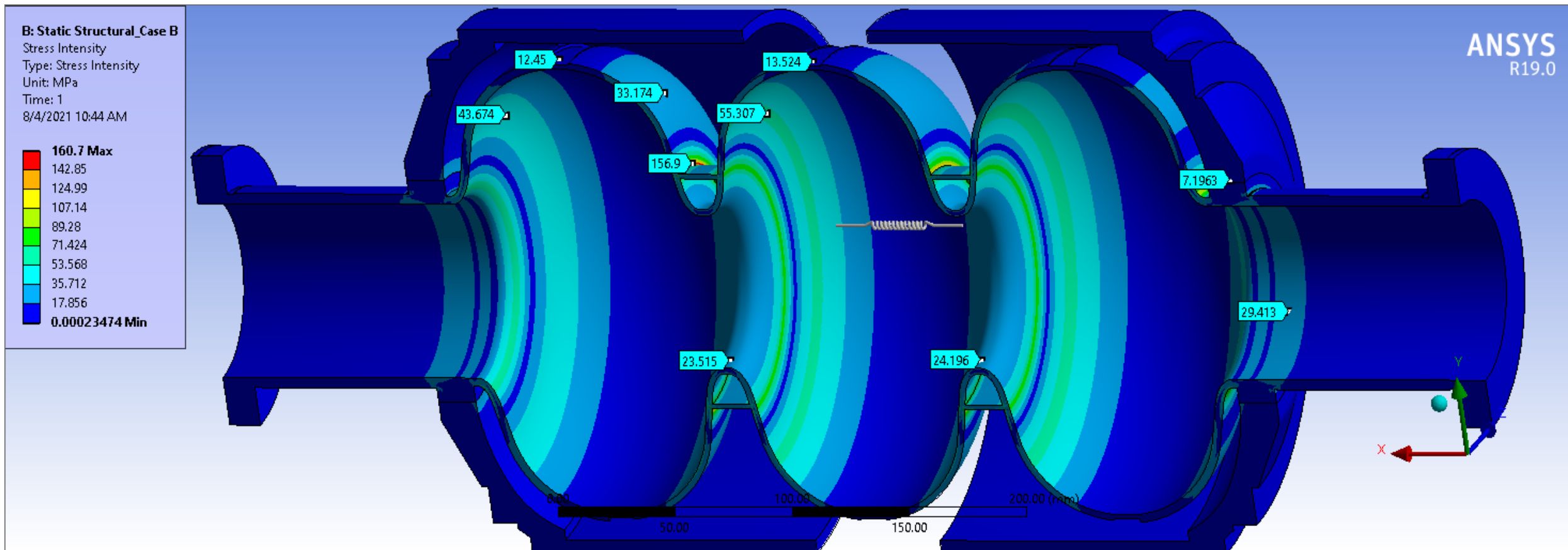
Solution - Case A

- Solution -> insert -> Stress intensity -> select bodies or surface to show
- Stress intensity is the largest of absolute value of difference in principle stresses ($\sigma_1 - \sigma_2$, $\sigma_2 - \sigma_3$, or $\sigma_3 - \sigma_1$).



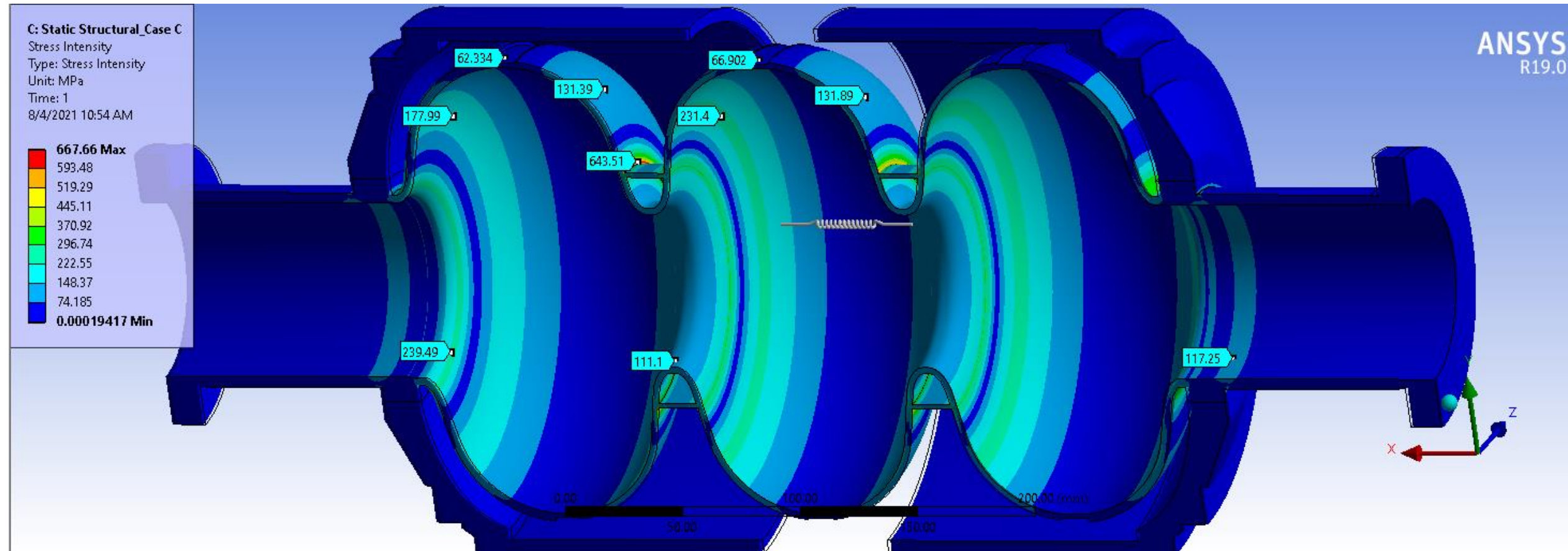
Solution - Case B

- Solution -> insert -> Stress intensity -> select bodies or surface to show
- Stress intensity is the largest of absolute value of difference in principle stresses ($\sigma_1 - \sigma_2$, $\sigma_2 - \sigma_3$, or $\sigma_3 - \sigma_1$).



Solution - Case C

- Solution -> insert -> Stress intensity -> select bodies or surface to show
- Stress intensity is the largest of absolute value of difference in principle stresses ($\sigma_1 - \sigma_2$, $\sigma_2 - \sigma_3$, or $\sigma_3 - \sigma_1$).



Solution: Maximum Stress Intensity

Case	Nb Half Cells [MPa]	Ti Tank [MPa]	Nb-Ti weld [MPa]
A	70	5	12
B	76	5	23
C	300	18	65

Case	Stiffener Ring Weld [MPa]	Iris Weld [MPa]	Equator Region weld [MPa]
A	147	22	12
B	161	24	14
C	668	111	67

- For the high-pressure gas safety regulations, the mechanical properties of the materials should be > 1.5 times the maximum stress intensity for 0.2% Y.S, and > 4 times the maximum stress intensity for T.S, on individual components.
- Data is different for 9-Cell SRF cavity structure, the stress values are lower.

Eigenvalue Buckling Analysis

What is Buckling?



- It is a sudden change of shape of a structural component under compressive or its own load.
- It can occur even though the stresses that develop in the structure are well below those needed to cause failure in the material.
- Further loading may cause significant and somewhat unpredictable deformations.
- Excess loading might or might not lead to complete loss of the structure's load-carrying capacity.

Buckled Structures

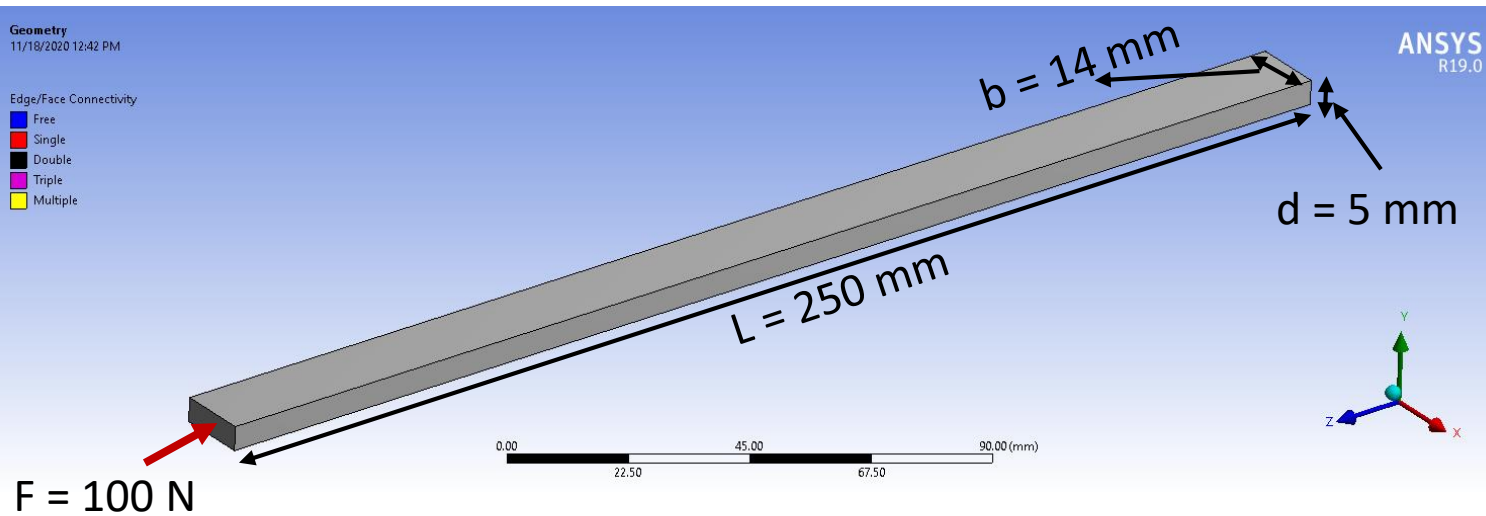


<https://www.structuresinsider.com/post/the-difference-between-buckling-compression-shear>



<https://www.volpe.dot.gov/infrastructure-systems-and-technology/structures-and-dynamics/track-buckling-research>

Buckling of a Solid Column

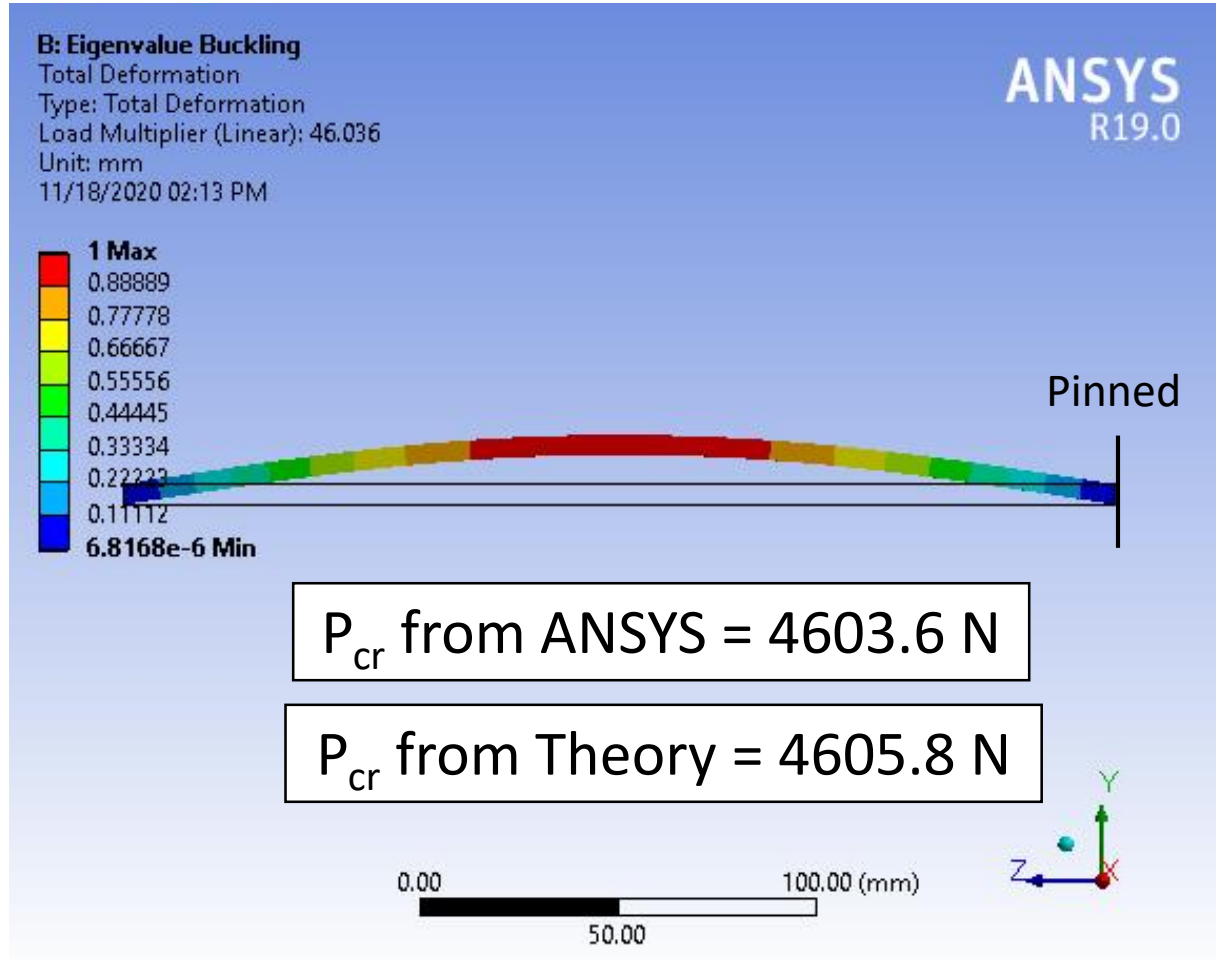


$$\text{Euler's Critical Load, } P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

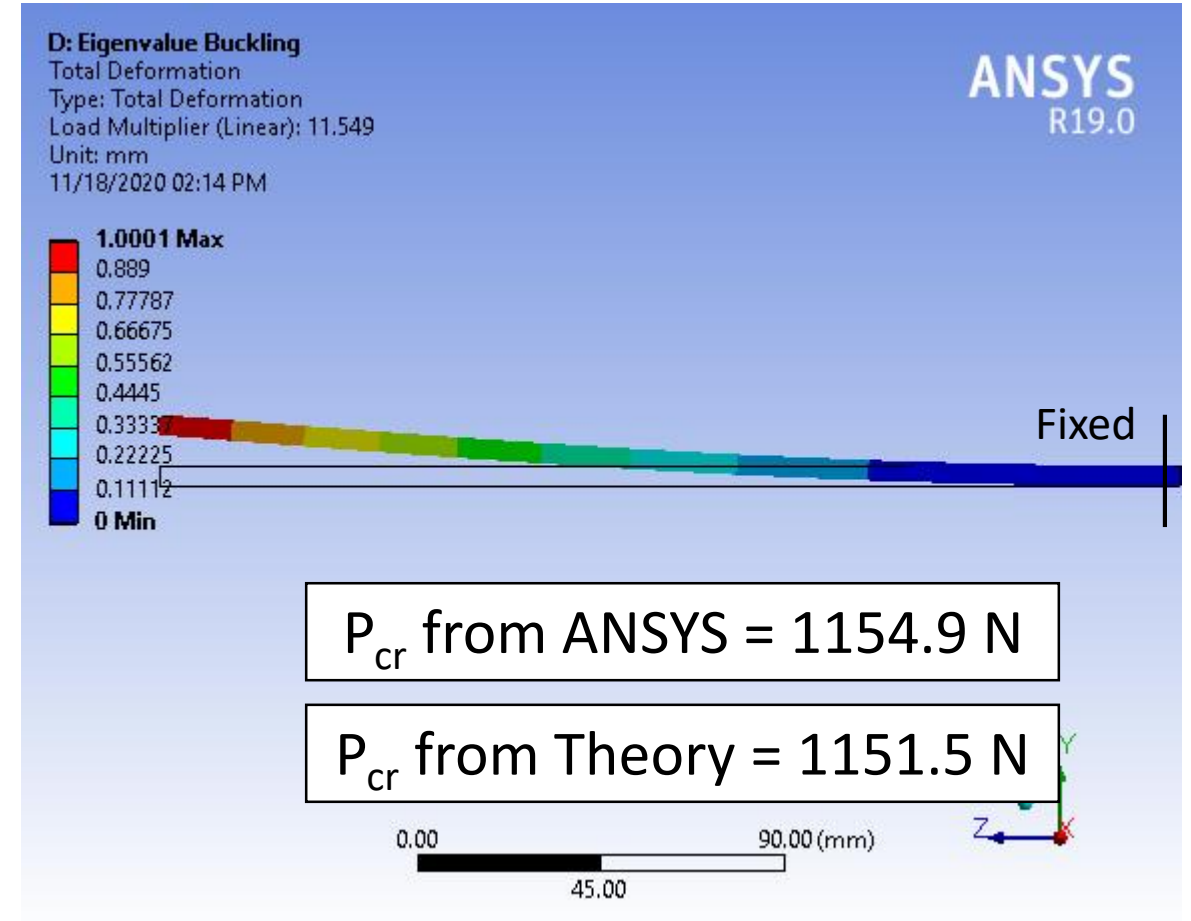
- Where 'E' is Young's modulus, 'I' is moment of inertia, 'K' is column effective length factor.
- Stainless Steel $E = 200 \text{ GPa}$, $I_{xx} = b \cdot d^3 / 12$

Buckled shape of column shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value K	0.65	0.80	1.2	1.0	2.10	2.0
End condition key						
	Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free					

Results – First Mode of buckling

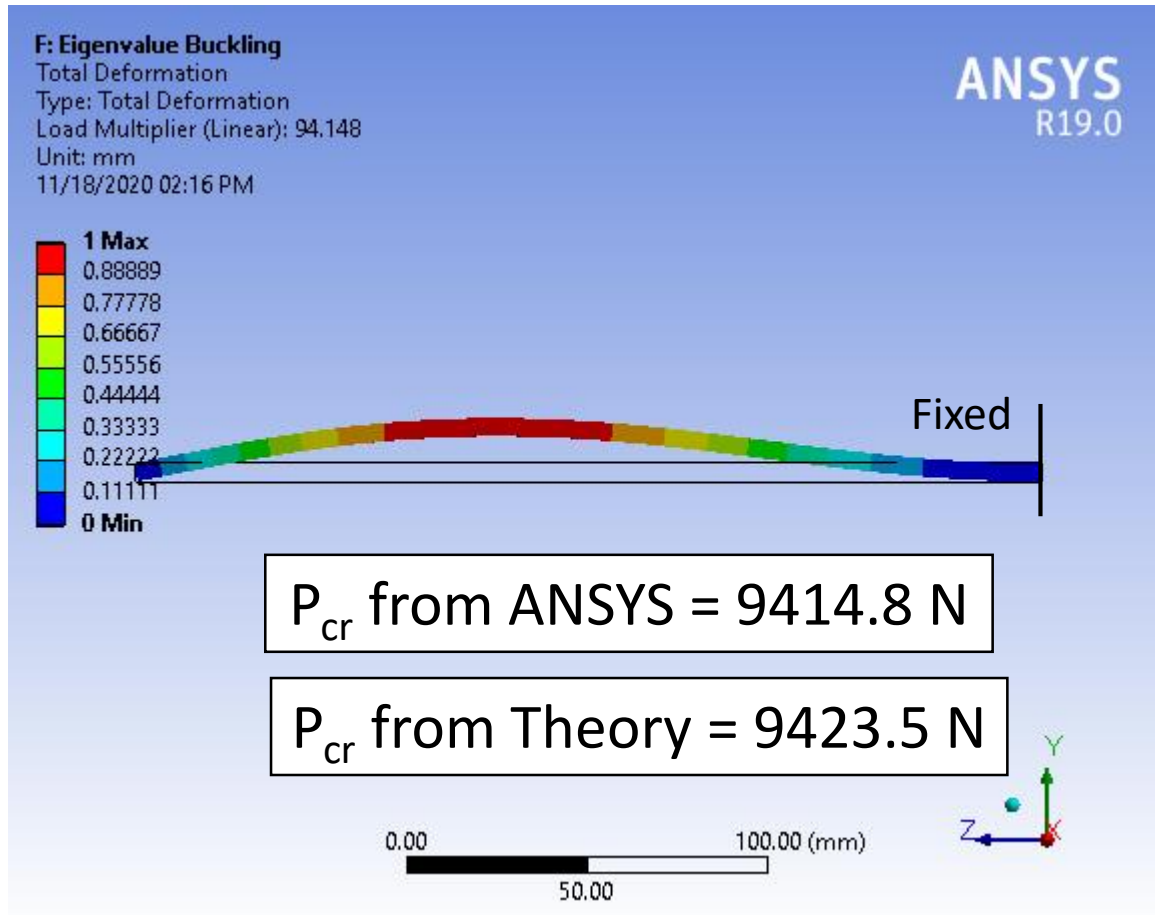


Pinned-Pinned Column

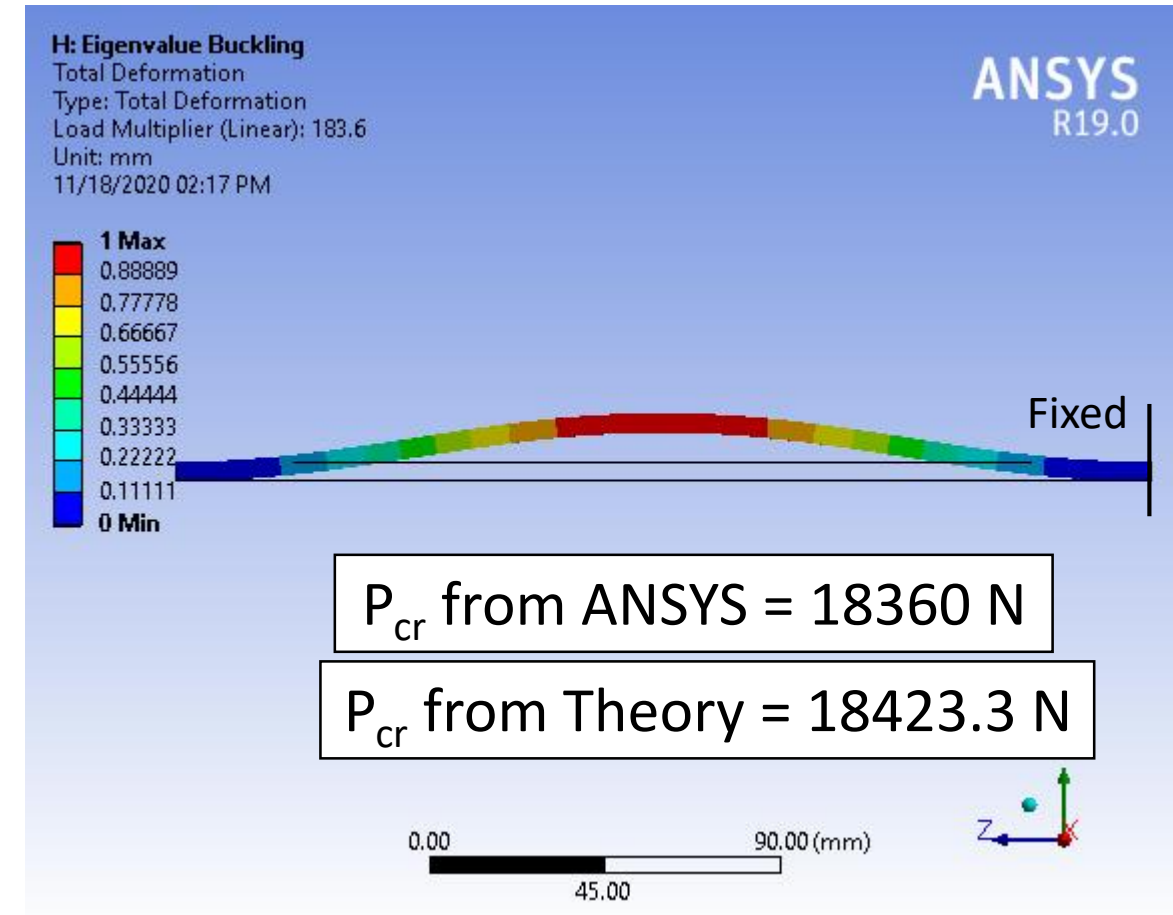


Fixed-Free Column

Results – First Mode of buckling



Fixed-Pinned Column

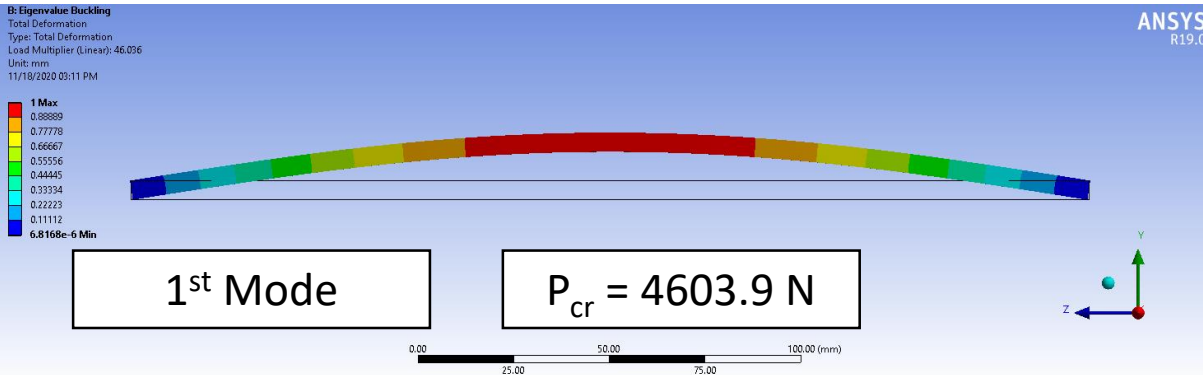


Fixed-fixed Column

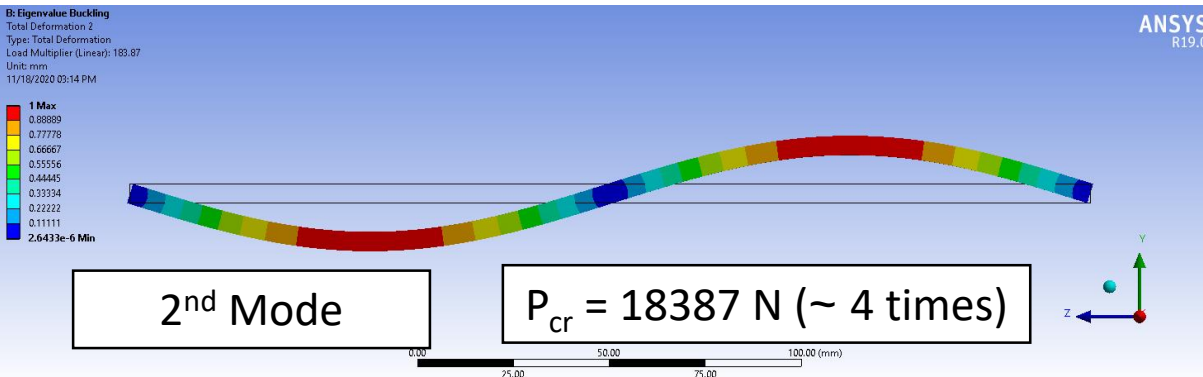
Modes of Pinned-Pinned Column Buckling



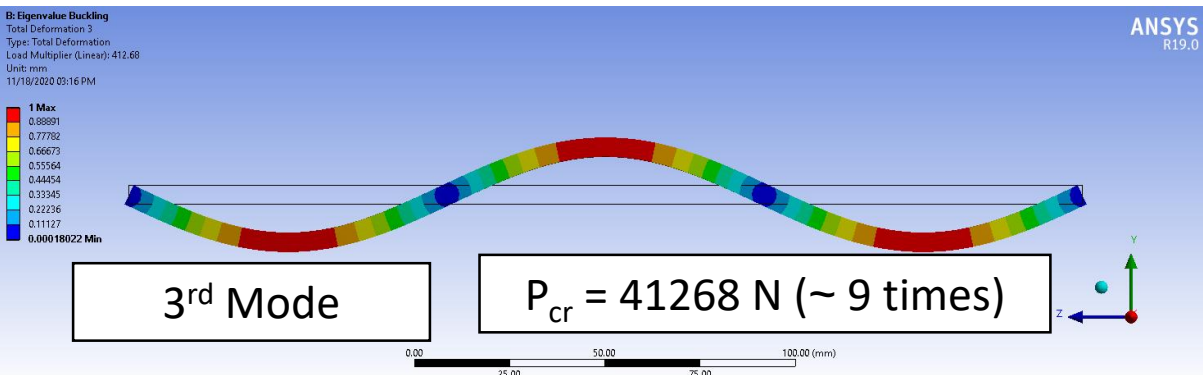
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First mode of buckling $P_1 = \frac{\pi^2 EI}{L^2}$



Second mode of buckling $P_2 = \frac{4\pi^2 EI}{L^2}$



Third mode of buckling $P_3 = \frac{9\pi^2 EI}{L^2}$

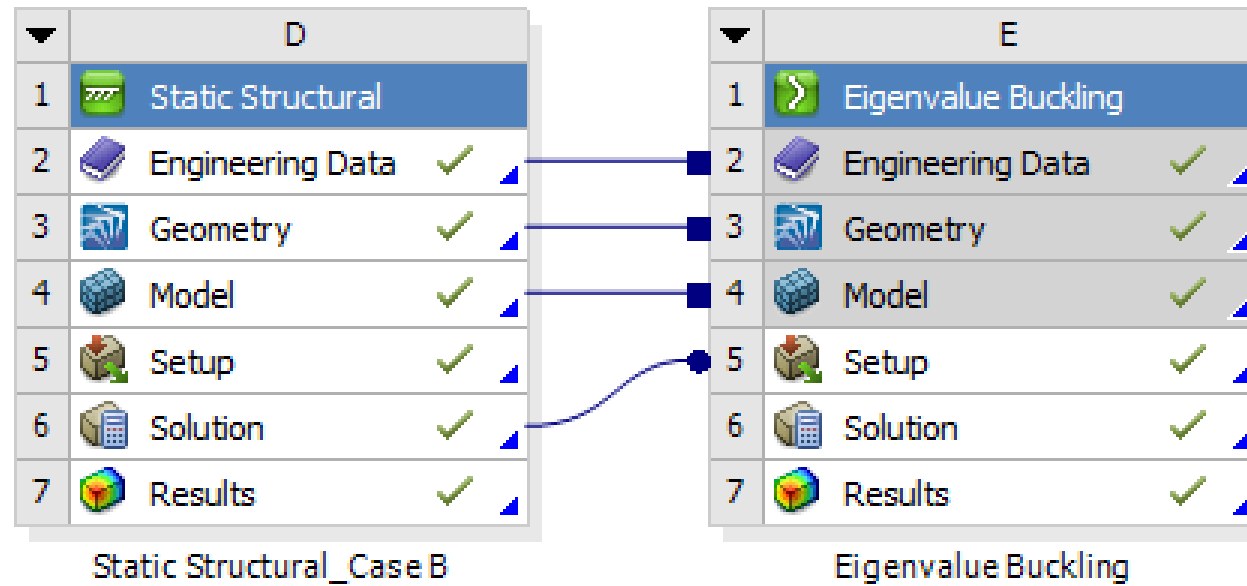
Eigen value Buckling Analysis for 1.3 GHz SRF Cavity

Eigenvalue Buckling of SRF Cavity



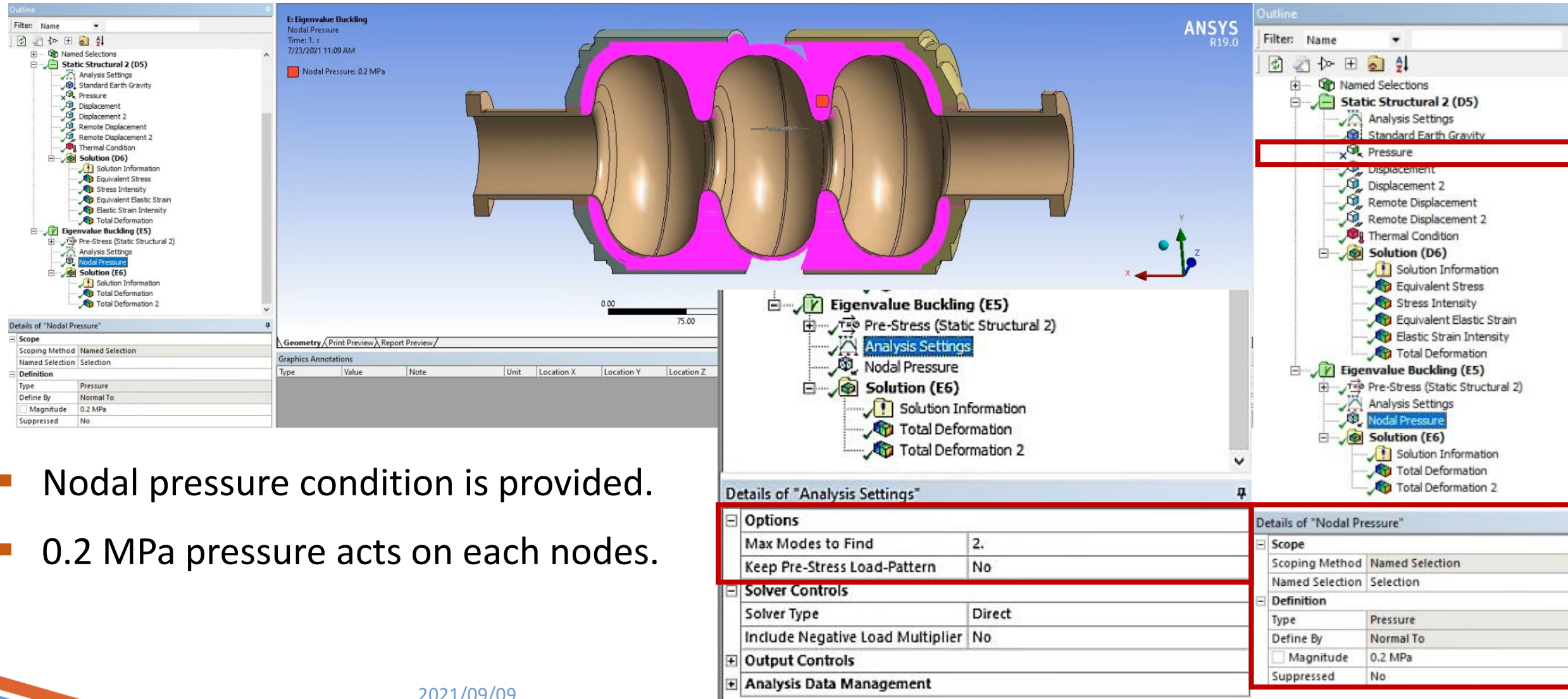
- Goal is to determine the pressure at which the cavity structure will buckle.
- Here only two Eigen modes will be determined.
- For this analysis, Case B is taken into consideration for buckling analysis.
- Cavity structure is easier to buckle in Case B rather than Case C due to lower tuner displacement.

Case Structure



- Same Geometry is transferred by linking.
- Model with the meshing is also linked.
- Setup is transferred, although pressure boundary condition is suppressed.

Eigenvalue Buckling – Analysis Setting



Outline

- Named Selections
- Static Structural 2 (D5)
 - Analysis Settings
 - Standard Earth Gravity
 - Pressure
 - Displacement
 - Displacement 2
 - Remote Displacement
 - Remote Displacement 2
 - Thermal Condition
- Solution (D6)
 - Solution Information
 - Equivalent Stress
 - Stress Intensity
 - Equivalent Elastic Strain
 - Elastic Strain Intensity
 - Total Deformation
- Eigenvalue Buckling (E5)
 - Pre-Stress (Static Structural 2)
 - Analysis Settings
 - Nodal Pressure
 - Solution (E6)
 - Solution Information
 - Total Deformation
 - Total Deformation 2

Details of "Nodal Pressure"

Scope	
Scoping Method	Named Selection
Named Selection	Selection

Definition	
Type	Pressure
Define By	Normal To
Magnitude	0.2 MPa
Suppressed	No

Details of "Analysis Settings"

Options	
Max Modes to Find	2.
Keep Pre-Stress Load-Pattern	No

Solver Controls	
Solver Type	Direct
Include Negative Load Multiplier	No

Output Controls	
Analysis Data Management	

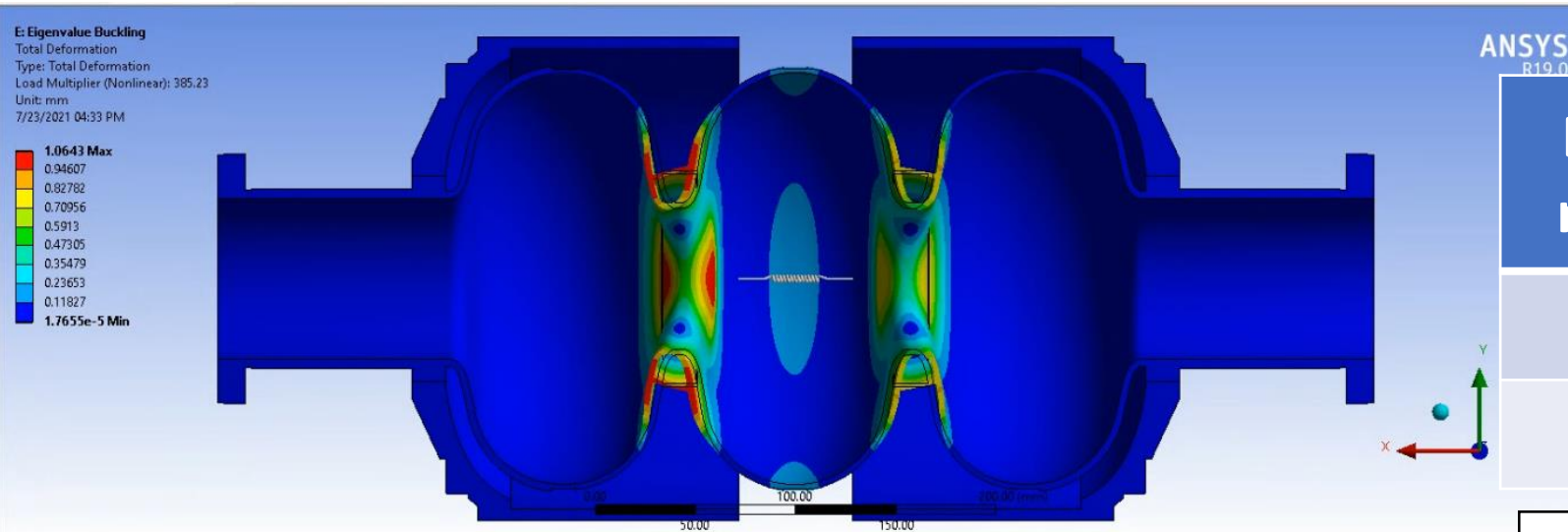
Details of "Nodal Pressure"

Scope	
Scoping Method	Named Selection
Named Selection	Selection

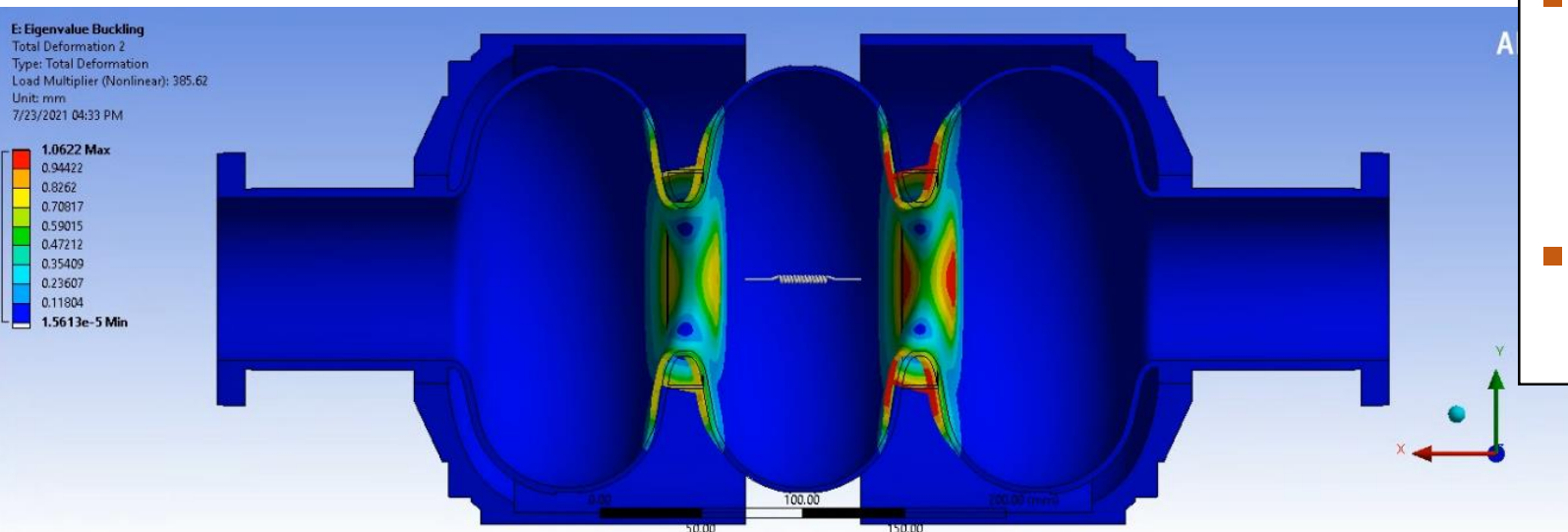
Definition	
Type	Pressure
Define By	Normal To
Magnitude	0.2 MPa
Suppressed	No

- Nodal pressure condition is provided.
- 0.2 MPa pressure acts on each nodes.

Solution



Eigen mode	Load Multiplier	Buckling Pressure
1 st	385.2	77.04
2 nd	386.6	77.32



- Eigenvalue buckling pressure is higher than the requirement ($> 4 \times 0.2$ MPa).
- For 9-Cell SRF cavity it has been calculated to be 33 MPa.

Non-linear Structure Analysis

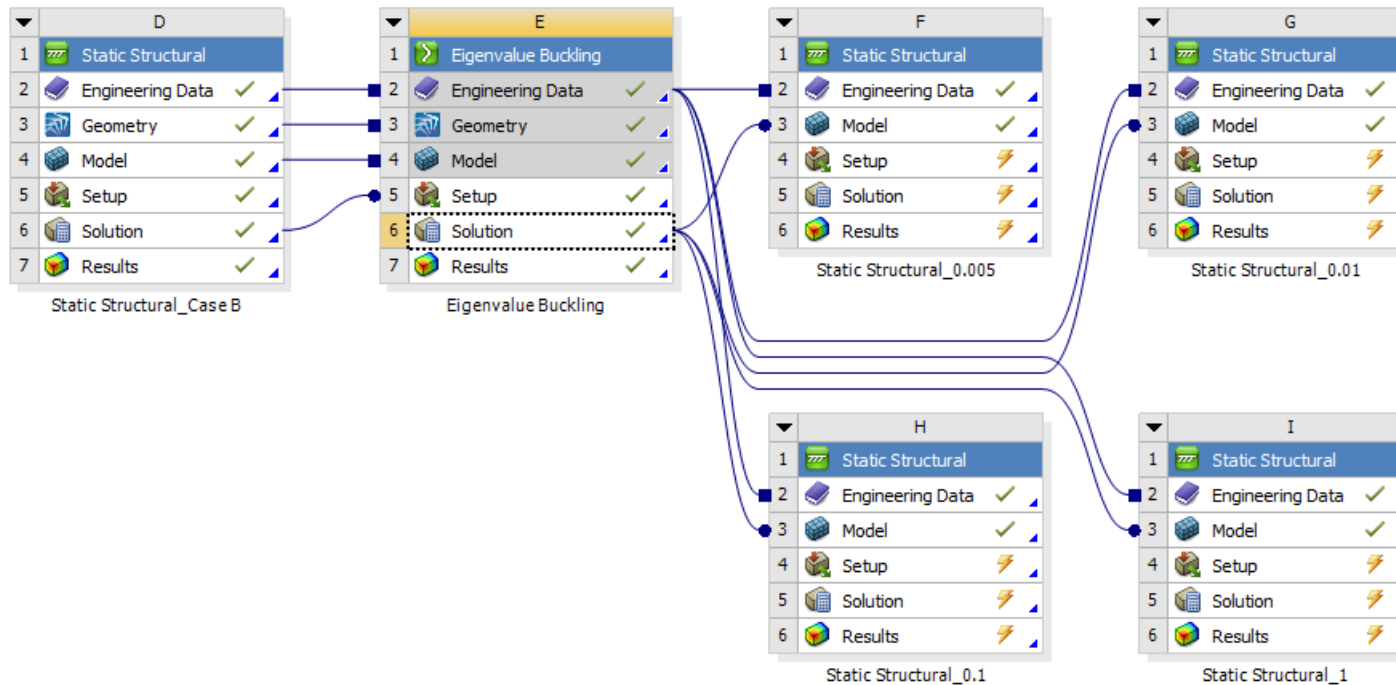
Non-linear Buckling Analysis

- Continuation of the Eigen value buckling analysis.
- Buckled structure can be transferred for a static structure analysis.
- Possible to control the level of buckling in the structure that would be transferred with *scale factor*, which can be varied from 0 to > 1 .
- Original structure scale factor is 0 and 1 being the buckled structure that would be transferred to the static structure analysis.
- Effect of structural deformities on buckling pressure can be determined.
- 1st Eigen mode was transferred to four static structure analysis studies with the scale factors being varied from 0.005, 0.01, 0.1 and 1.

Non-linear Structure Analysis



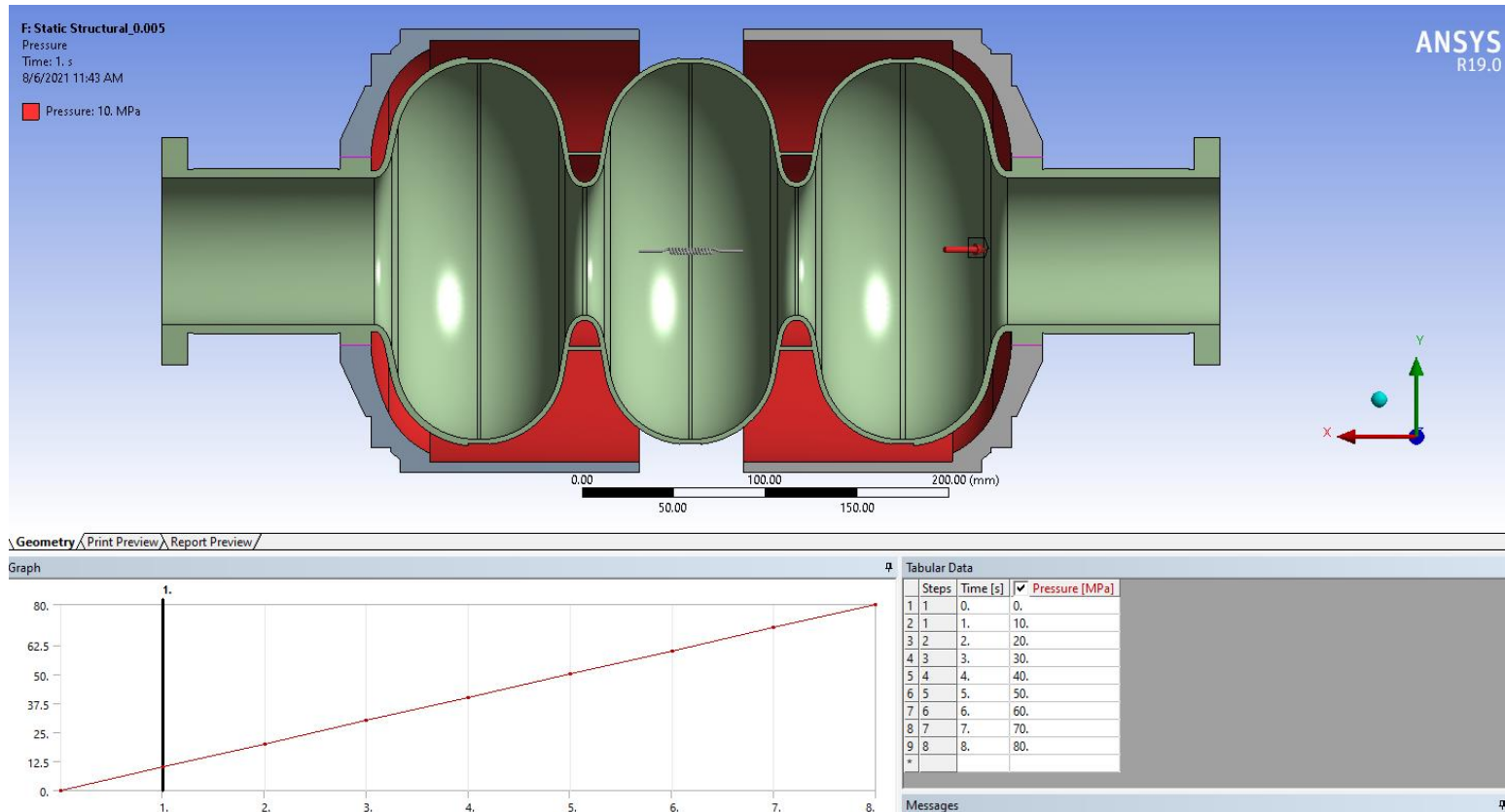
Center for
Applied
Superconducting
Accelerator
応用超伝導加速器センター



- 1st Eigen mode was transferred to four static structure analysis studies with the scale factors varied from 0.005, 0.01, 0.1 and 1.

14	Update Option	Use application default
15	Solve Process Setting	My Computer
16	Queue	
17	Update Settings for Static Structural_0.005 (Component ID: Model 5)	
18	Process Nodal Components	☒
19	Nodal Component Key	
20	Process Element Components	☒
21	Element Component Key	
22	Scale Factor	0.005
23	Mode	1
24	Update Settings for Static Structural_0.01 (Component ID: Model 6)	
25	Process Nodal Components	☒
26	Nodal Component Key	
27	Process Element Components	☒
28	Element Component Key	
29	Scale Factor	0.01
30	Mode	1
31	Update Settings for Static Structural_0.1 (Component ID: Model 7)	
32	Process Nodal Components	☒
33	Nodal Component Key	
34	Process Element Components	☒
35	Element Component Key	
36	Scale Factor	0.1
37	Mode	1
38	Update Settings for Static Structural_1 (Component ID: Model 8)	
39	Process Nodal Components	☒
40	Nodal Component Key	
41	Process Element Components	☒
42	Element Component Key	
43	Scale Factor	1
44	Mode	1

Setup – Boundary Condition



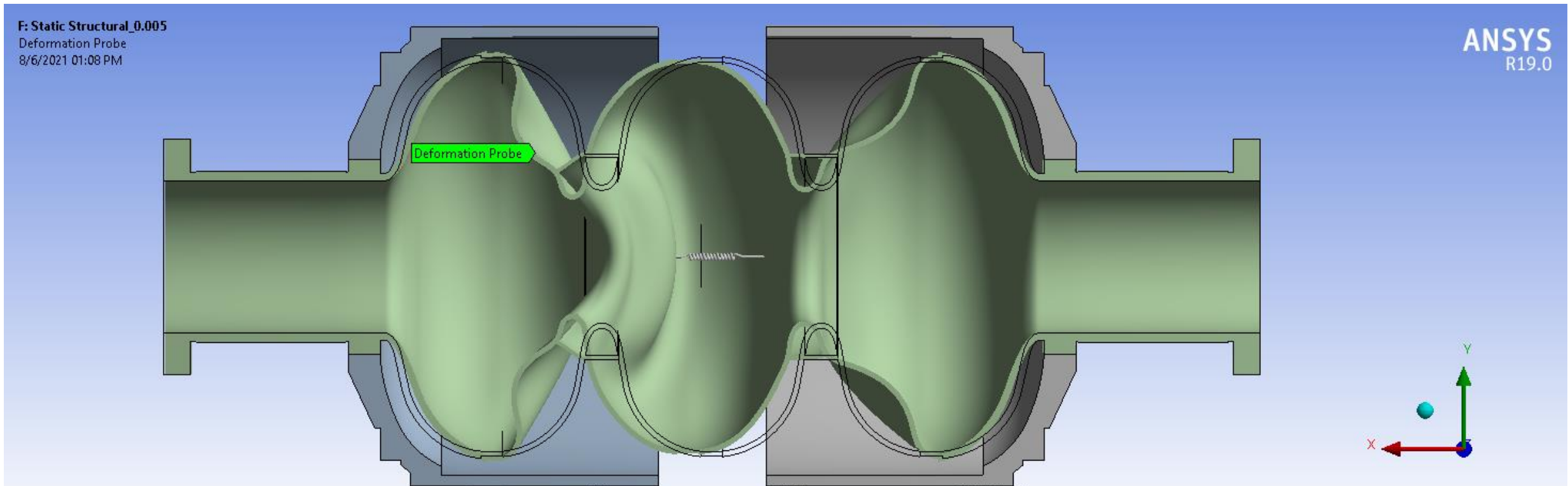
Tabular Data

	Steps	Time [s]	☒ Pressure [MPa]
1	1	0.	0.
2	1	1.	10.
3	2	2.	20.
4	3	3.	30.
5	4	4.	40.
6	5	5.	50.
7	6	6.	60.
8	7	7.	70.
9	8	8.	80.
*			

Messages

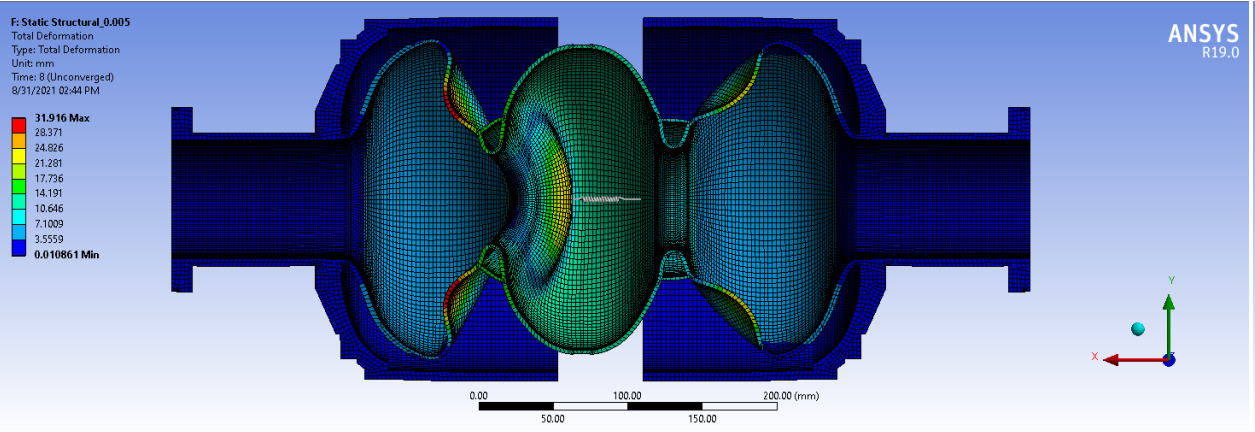
- Boundary conditions remains the same as in Case B, except pressure.
- Pressure (faces in Red) is increased incrementally in steps.
- Pressure increased from 0 – 80 MPa in 8 steps.

Solution – Buckling Pressure

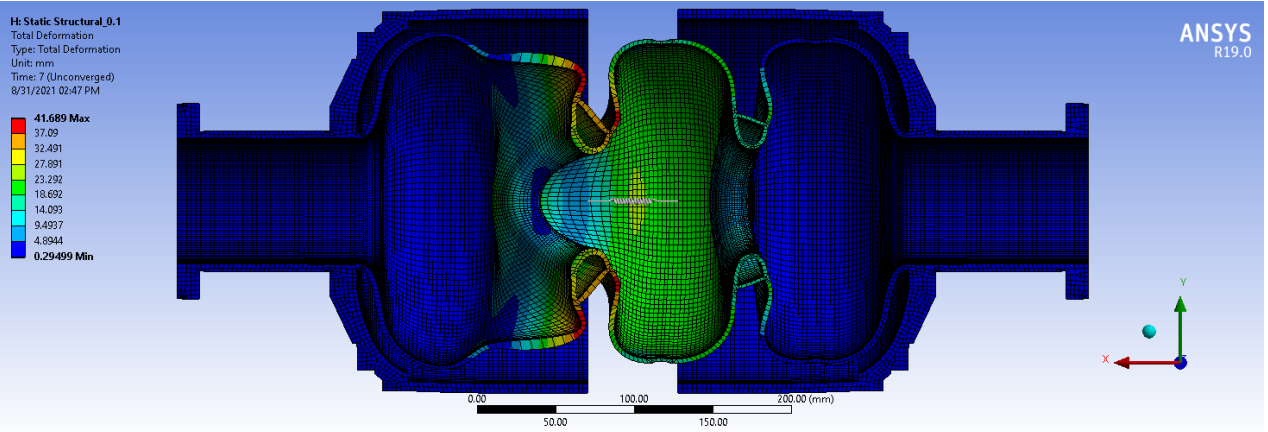


- Buckling pressure was determined by placing a total deformation probe on a point on the cavity structure where the maximum buckling occurs.
- The total deformation obtained from the deformation probe was plotted against the pressure condition for various scaling factors.

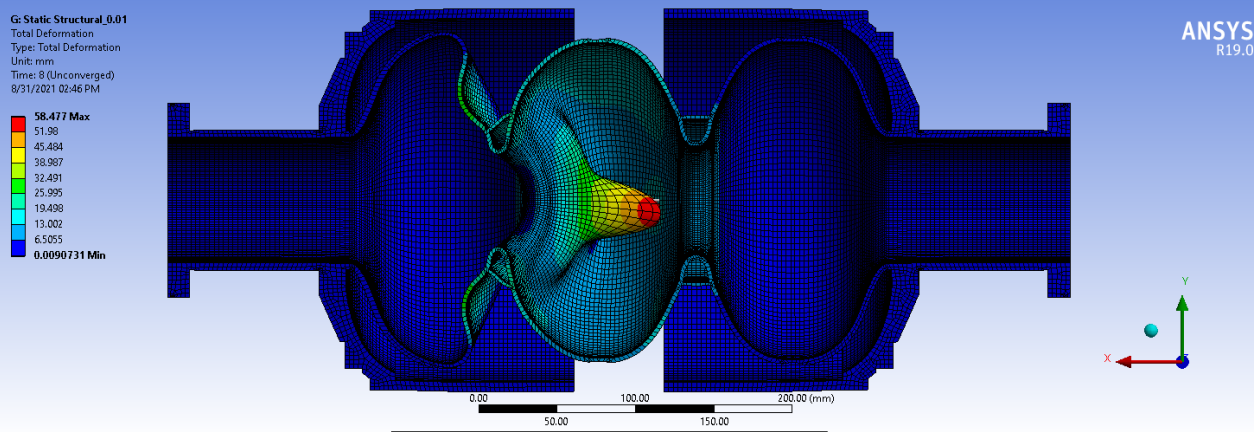
Buckled Structures for Scale factors



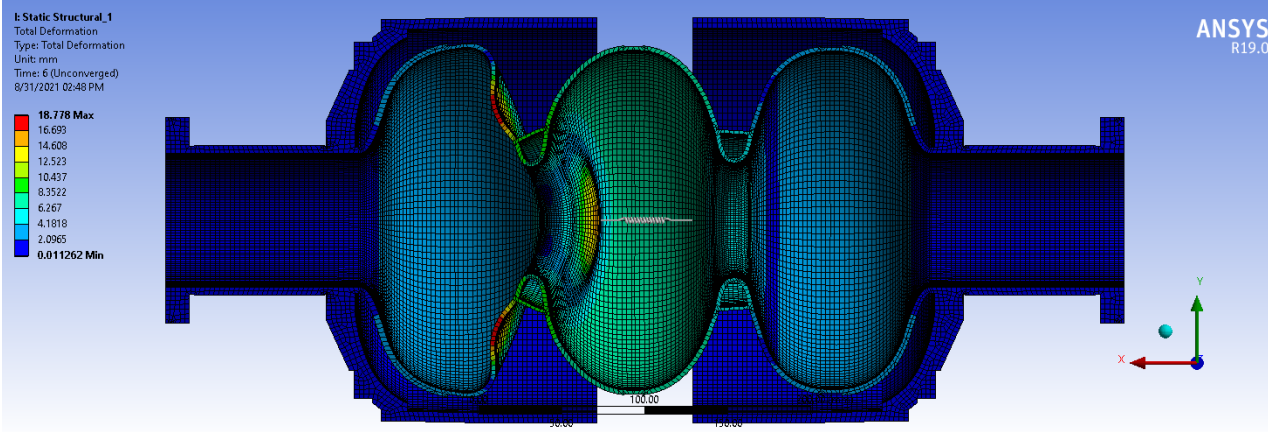
Scale Factor – 0.005



Scale Factor – 0.1



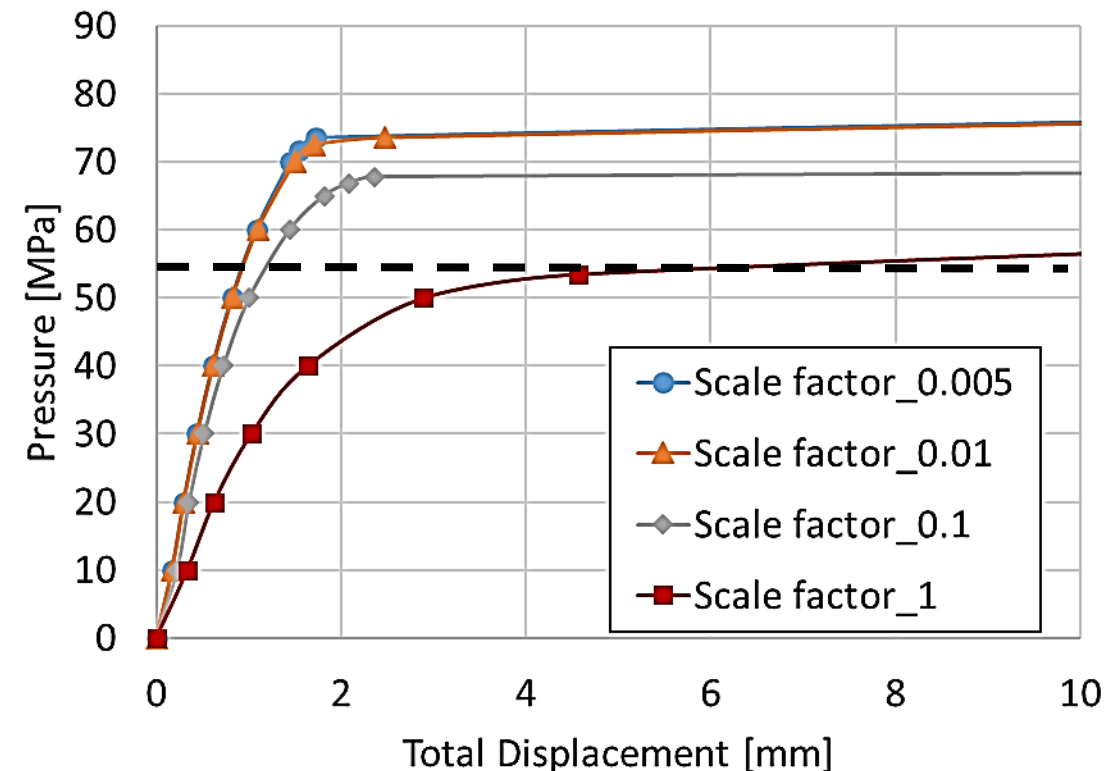
Scale Factor – 0.01



Scale Factor – 1

Results and Discussions

- Buckling pressure was 74 MPa for scale factor of 0.005, similar to Eigenvalue buckling analysis.
- Buckling pressure reduces to 54 MPa for the scale factor of 1.
- For elastic material properties, the 3-Cell cavity will not buckle upto 54 MPa pressure.
- Buckling pressure usually should be > 4 times the MAWP, and in this case the cavity structure has sufficient strength to qualify for that criterion.



Thank you for your attention!