

Introduction to LLRF

OHO 高エネルギー加速器セミナー OHO'21

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Disclaimer



- This lecture will give only a rough overview, it is incomplete by its nature

Contents



- Introduction
- Cavity theory
- LLRF system overview
- Signal sampling
- Digital signal processing and implementation
- Controller theory
- Example features of an LLRF system
- Summary

Introduction

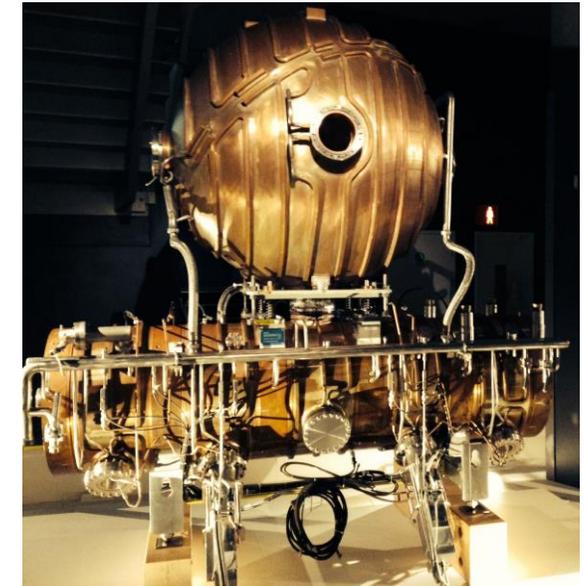
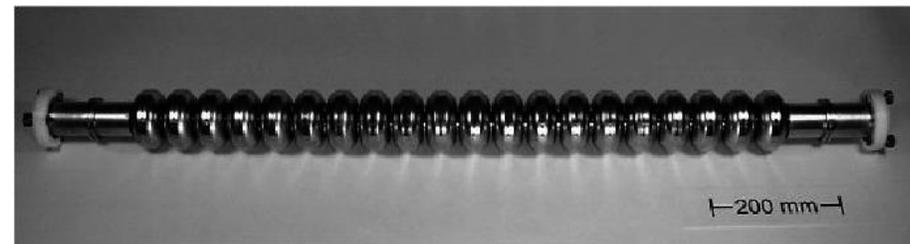
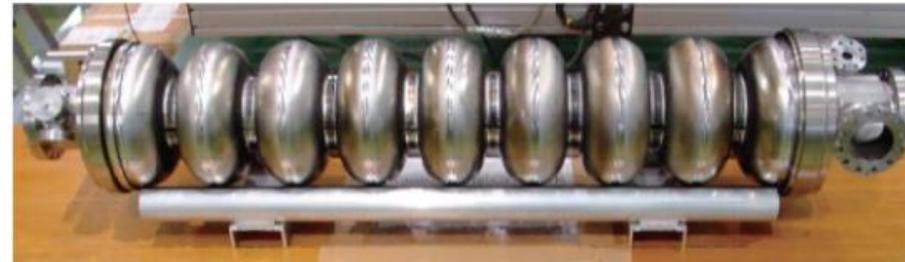
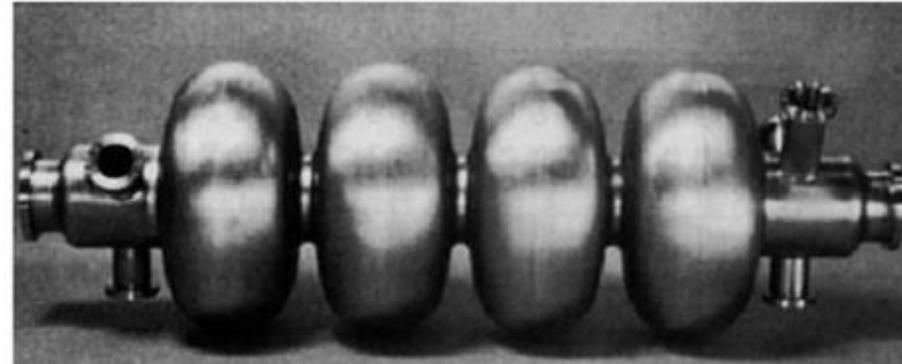
What does LLRF stand for? What is it about?



- Low Level Radio Frequency
- The goal: control the amplitude and phase of electro-magnetic fields within cavities
 - Required at a wide range of facilities, from small test facilities to large scale accelerators
- These fields can have high amplitudes and high frequencies
- Thus down-conversion to small amplitudes for detection is applied
 - (and in some cases also a down-conversion to low frequencies, while preserving amplitude and phase information, is applied)

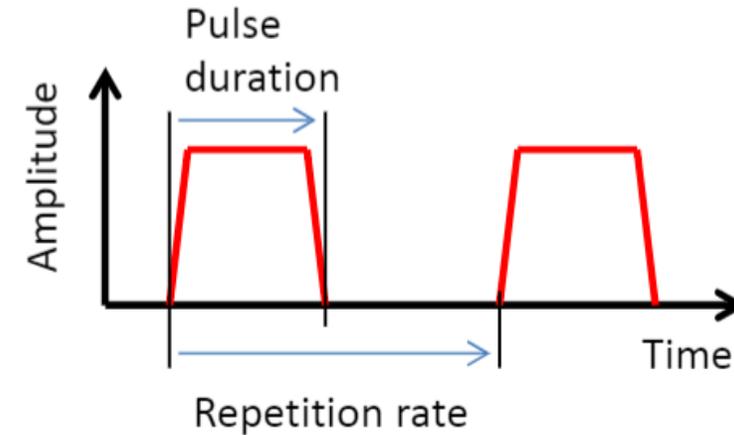
Superconducting and Normal Conducting Cavities

- Frequency ranges from MHz to tens of GHz



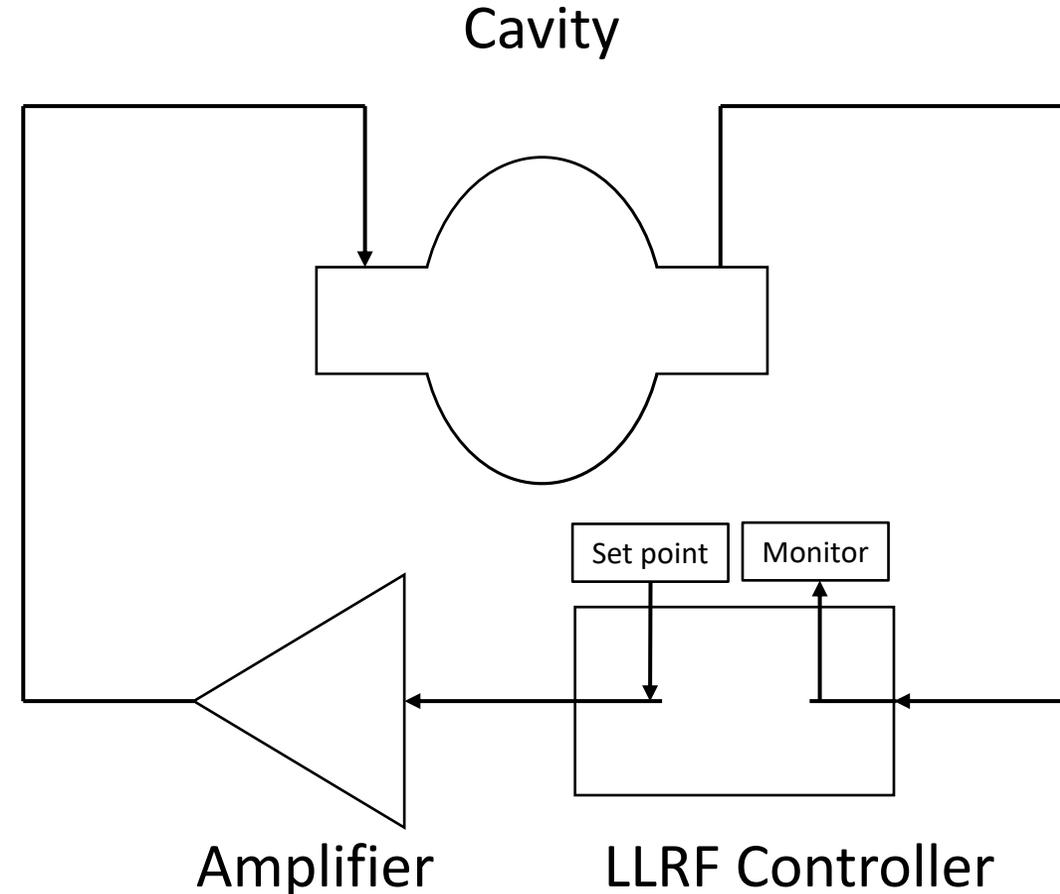
Modes of Operation

- Pulsed mode
 - Short Pulse mode (SP)
 - Duty factor of e.g. 1%
 - Long Pulse mode (LP)
 - Duty factor of 10% to 50%
 - Only a certain portion of time is useable for beam acceleration
- Continuous Wave (CW)
 - Continuous RF field
 - Duty factor of 100%
 - Beam can be accelerated all the time



Most basic layout of an RF system

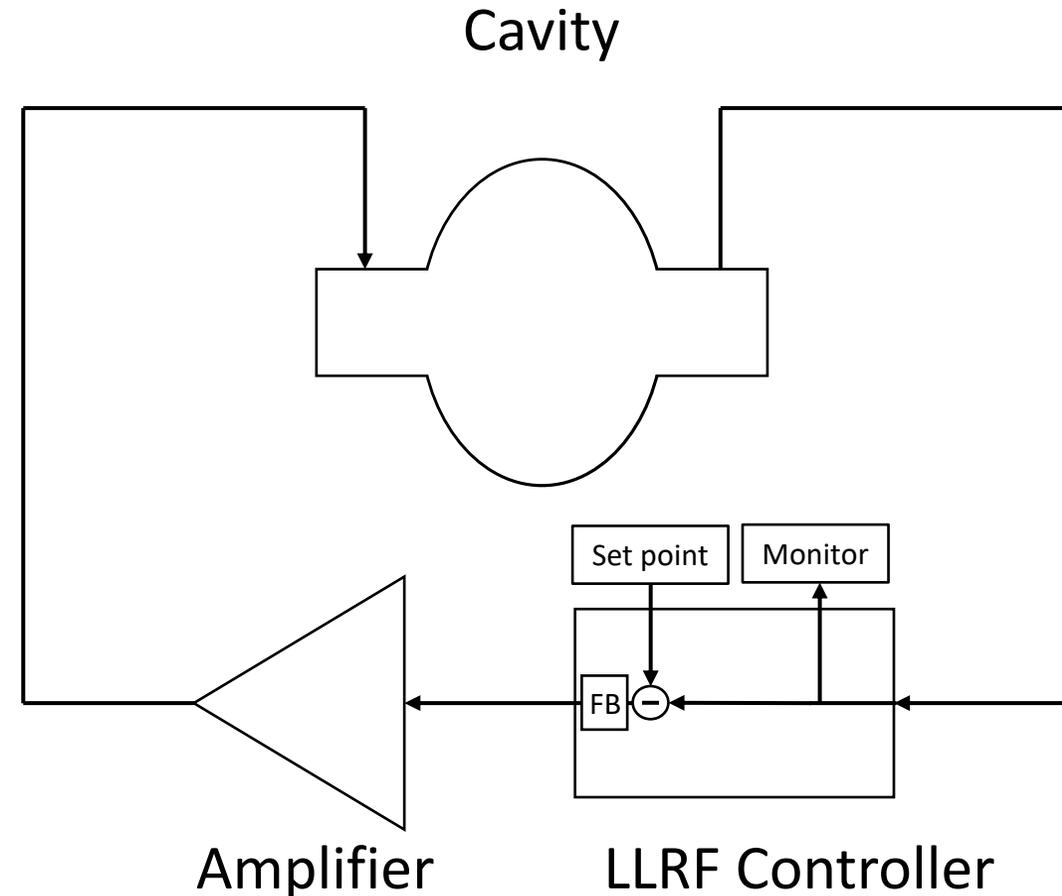
- Open loop operation
 - Controller creates drive signal corresponding to a set point
 - Signal is amplified
 - Signal is coupled into the cavity
 - Signal is coupled out of the cavity
 - Signal is detected by the controller



Most basic layout of an RF system

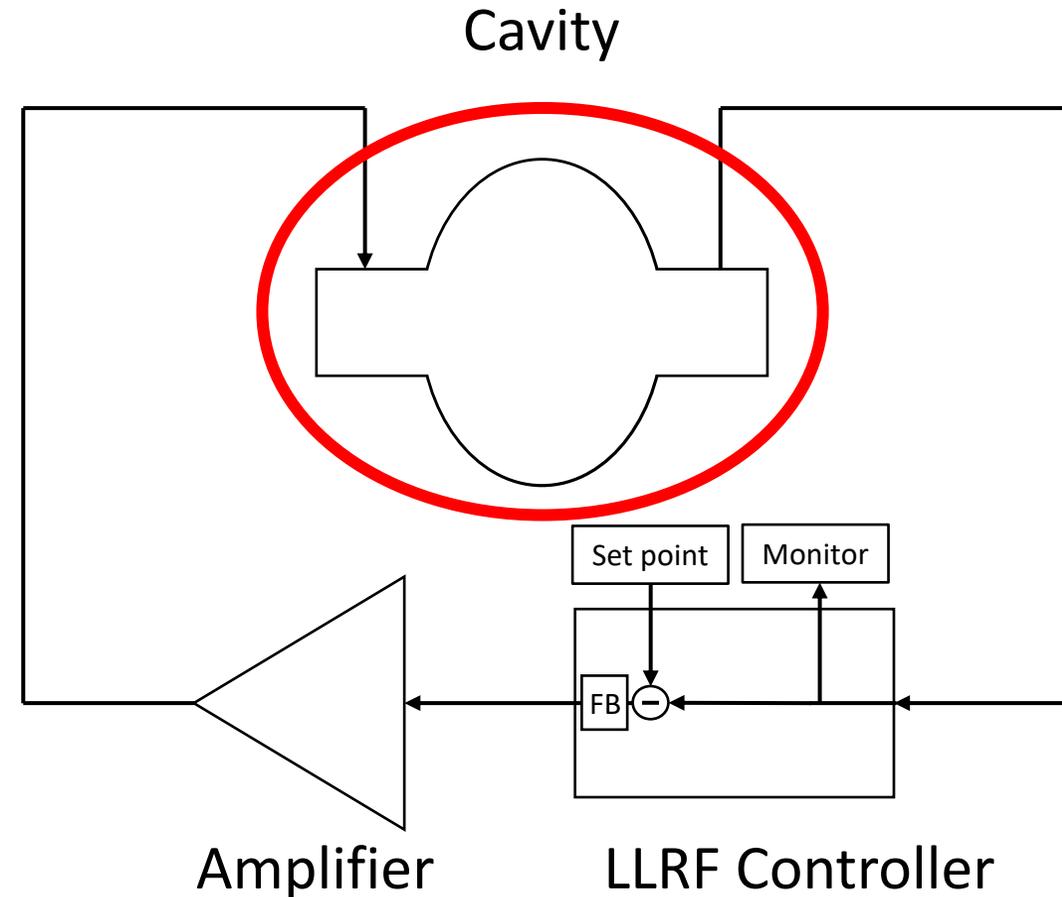
- Closed loop operation

- Controller creates drive signal corresponding to a set point
- Signal is amplified
- Signal is coupled into the cavity
- Signal is coupled out of the cavity
- Signal is detected by the controller
- Controller compares signal to the set point and adjusts the drive signal accordingly



Most basic layout of an RF system

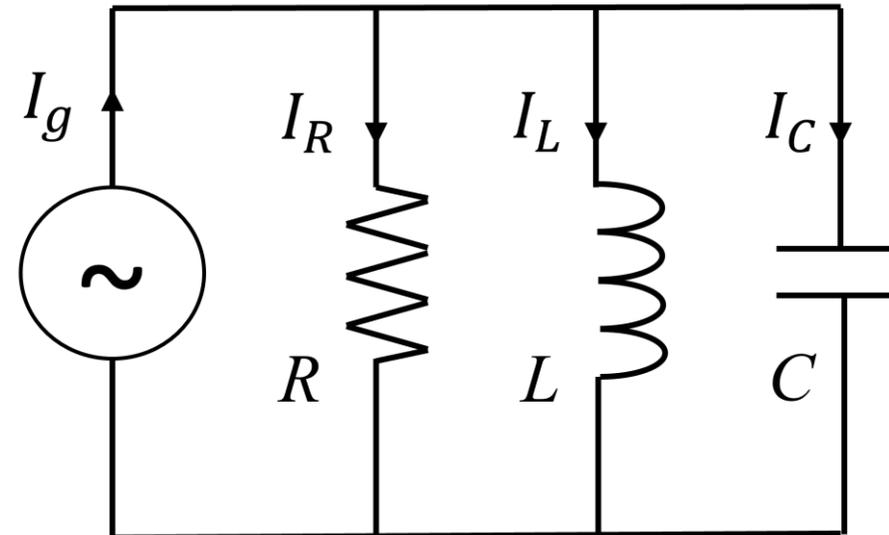
- Let's take a look at the cavity first



Cavity Theory

Cavity modeling: RCL model

- Electric circuit
 - Resistor R
 - Inductor L
 - Capacitor C
- Forms a harmonic oscillator



Cavity modeling: Quality factor in general

$$Q = 2\pi \frac{\text{stored energy in cavity}}{\text{dissipated energy per cycle}} = \frac{2\pi f_0 W}{P_{diss}}$$

Resonance frequency f_0 and Stored energy W point to the numerator, and Dissipated power P_{diss} points to the denominator.

Cavity modeling: Unloaded quality factor

- Assumes losses only due to surface resistance

Capacitance Square of amplitude of oscillating voltage

$$Q_0 = \frac{2\pi}{T} \cdot \frac{\frac{1}{2} CV_0^2}{\frac{1}{2} \frac{V_0^2}{R}}$$

Time period of an RF cycle Resistance

$W = \frac{1}{2} CV_0^2$
 $P_{diss} = \frac{V_0^2}{2R}$
 $\omega_0 = \frac{1}{\sqrt{LC}}$
 $\omega_0 = 2\pi f_0$

$Q_0 = \omega_0 RC = \frac{R}{L\omega_0} = \frac{\omega_0 W}{P_{diss}}$

Cavity modeling: External quality factor

- Accounts for external losses (e.g. via the power coupler)

$$Q_{ext} = 2\pi \frac{\text{stored energy in cavity}}{\text{dissipated energy in external devices per cycle}} = \frac{\omega_0 W}{P_{ext}}$$

↑
Dissipated power in
all external devices

Cavity modeling: Loaded quality factor

- Accounts for all losses

$$Q_L = 2\pi \frac{\text{stored energy in cavity}}{\text{total energy loss per cycle}} = \frac{\omega_0 W}{P_{tot}}$$

$$P_{tot} = P_{diss} + P_{ext}$$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

In case of SC cavities Q_0 is several orders of magnitude larger than Q_{ext} . Thus, Q_L is in the same order as Q_{ext} .

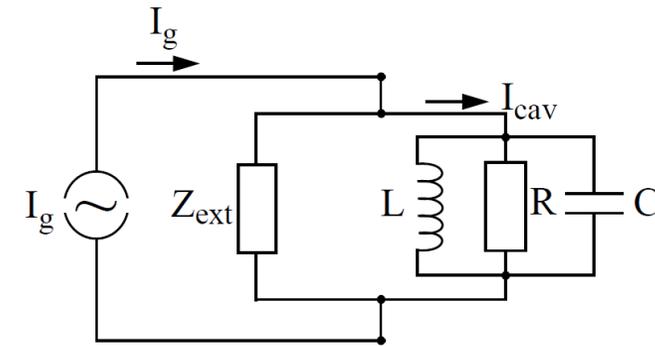
Cavity modeling: Definition of the Loaded Quality Factor

- Add transition line
- Impedance Z_{ext} is like a parallel resistor to R (characteristic impedance of a coaxial cable: 50Ω)
- Both can be replaced by the loaded shunt impedance R_L

$$\frac{1}{R_L} = \frac{1}{R} + \frac{1}{Z_{ext}}$$

$$Q_0 = \omega_0 RC = \frac{R}{L\omega_0} = \frac{\omega_0 W}{P_{diss}}$$

$$\frac{R}{Q_0} = \omega_0 = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$$



R/Q_0 depends only on ω_0 , C , and L , which means it depends only on the cavity geometry and not the surface resistance.

Cavity modeling: Definition of the Loaded Quality Factor

- The shunt impedance R_{sh} depends on the dissipated power
- Includes factor $\frac{1}{2}$ of the time average

$$P_{diss} = \frac{1}{2} \cdot \frac{V_{cav}^2}{R} = \frac{V_{cav}^2}{R_{sh}}$$

$$R = \frac{1}{2} R_{sh} = \frac{1}{2} \frac{r}{Q} Q_0$$

- Definition of normalized shunt impedance

$$\frac{r}{Q} := \frac{R_{sh}}{Q_0} = \frac{2R}{Q_0}$$

Cavity modeling: Definition of the Loaded Quality Factor

- Coupling between cavity and transmission line

$$\beta = \frac{R}{Z_{ext}}$$

$$\frac{1}{R_L} = \frac{1}{R} + \frac{1}{Z_{ext}}$$

$$R_L = \frac{R}{1 + \beta}$$

$$\frac{r}{Q} := \frac{R_{sh}}{Q_0} = \frac{2R}{Q_0}$$

$$Q_L = \frac{Q_0}{1 + \beta}$$

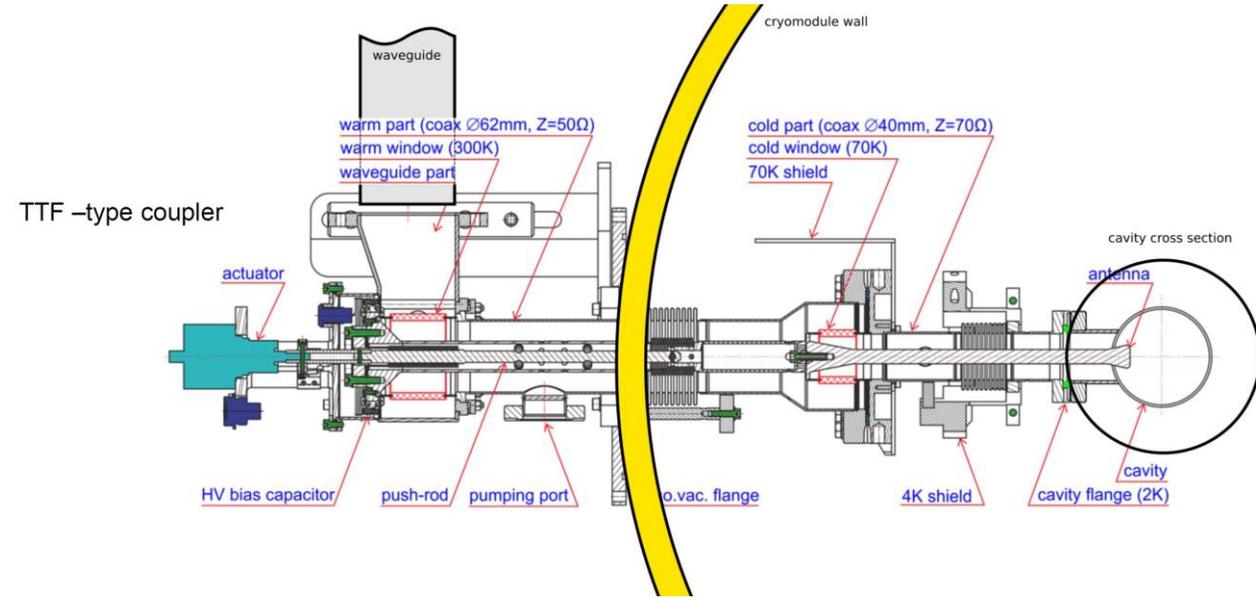
$$\omega_{1/2} = \frac{\omega_0}{2Q_L}$$

Q_L can be manipulated by changing the coupling β

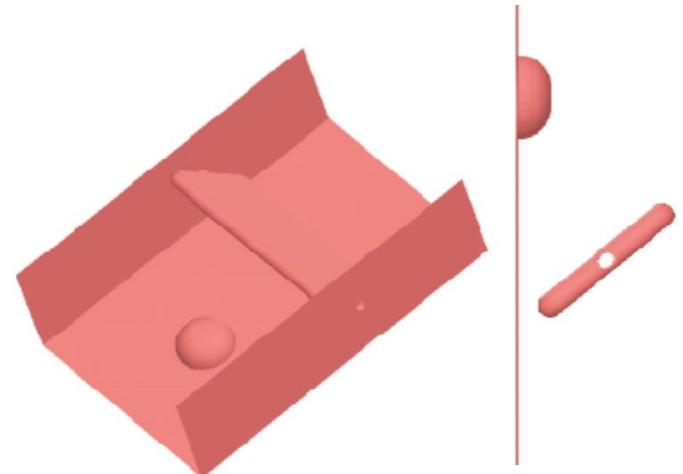
And with this the cavity bandwidth

Solutions for Changing the Coupling

- Change input depth via movable input coupler antenna

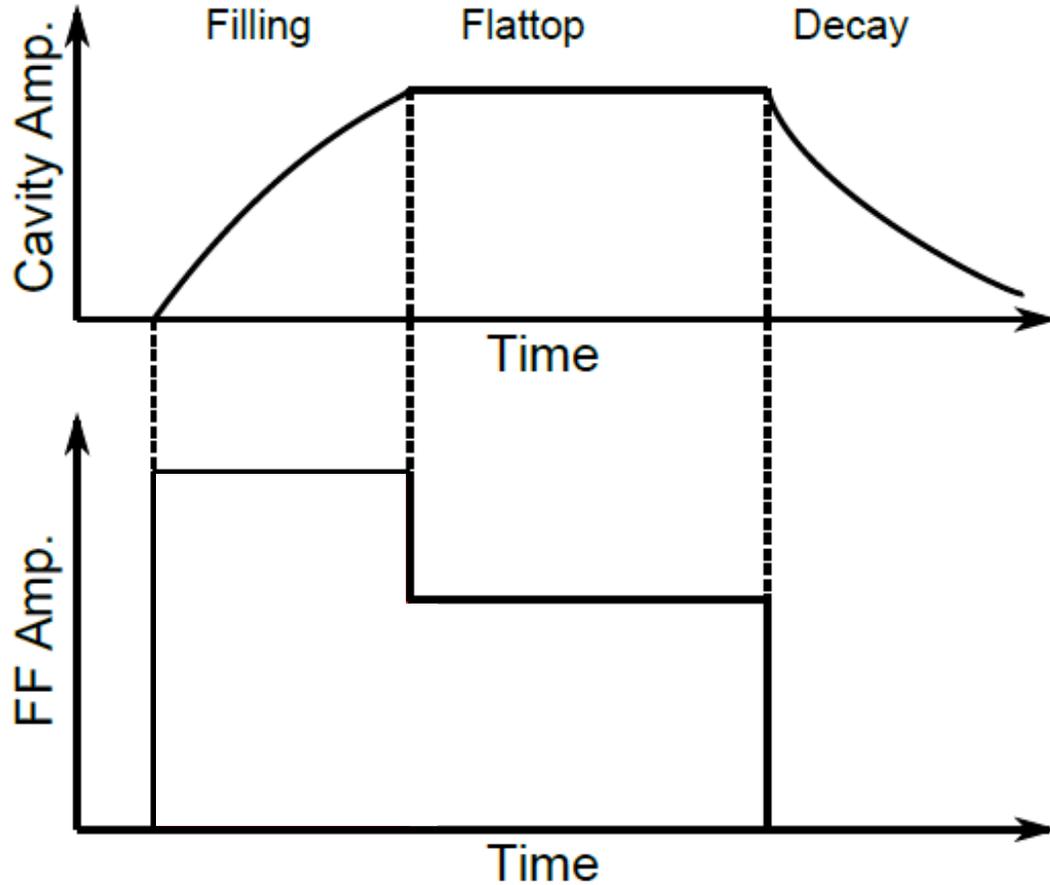


- Change angle of plate of waveguide reflector

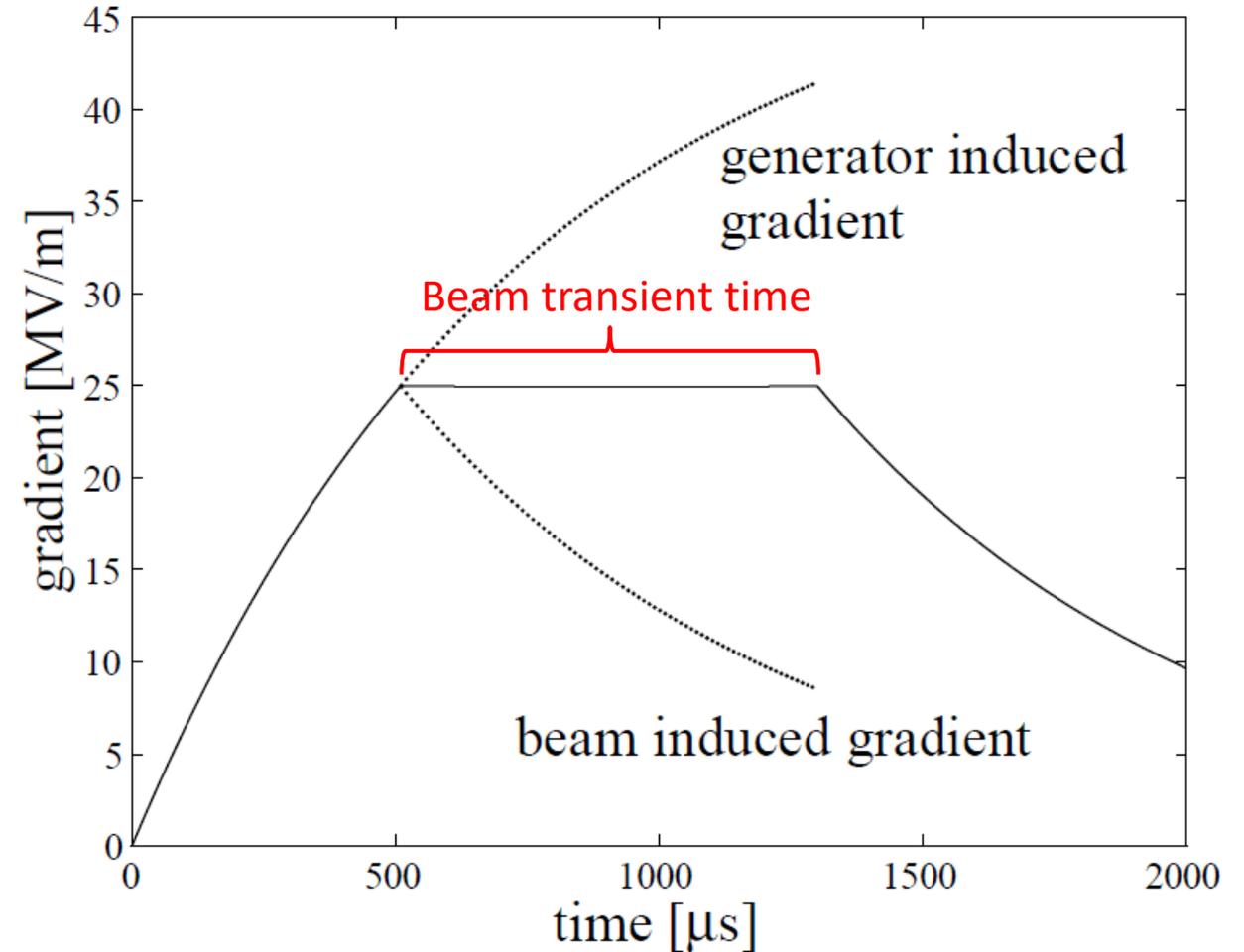


Pulsed Operation with Beam Loading

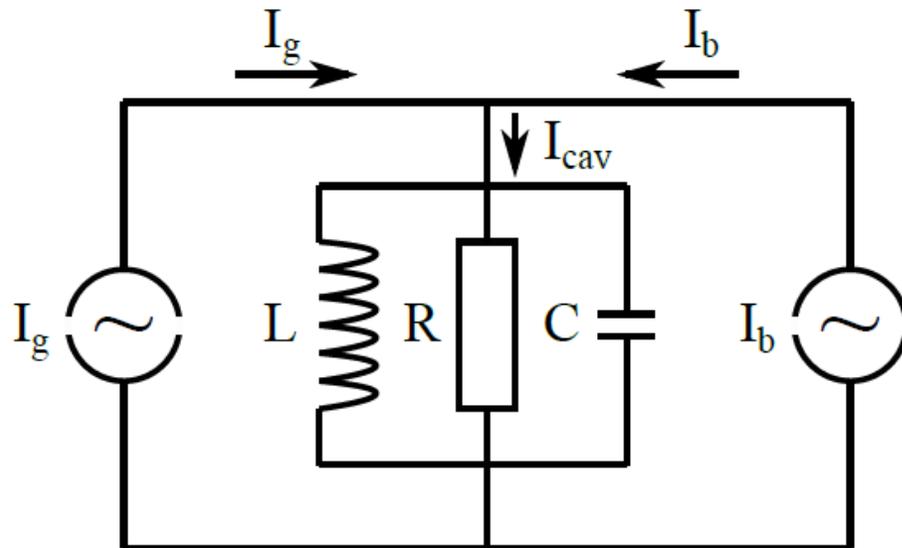
- Without beam



- With beam



Derivation of Filling and Flattop Powers



$$I_C + I_R + I_L = I_{cav}$$

$$\dot{I}_C + \dot{I}_R + \dot{I}_L = \dot{I}_{cav}$$

$$\dot{I}_C = C\ddot{V}_{cav}$$

$$\dot{I}_R = \frac{1}{R_L}\dot{V}_{cav}$$

$$\dot{I}_L = \frac{1}{L}V_{cav}$$

$$C\ddot{V}_{cav} + \frac{1}{R_L}\dot{V}_{cav} + \frac{1}{L}V_{cav} = \dot{I}_{cav}$$

Derivation of Filling and Flattop Powers

$$\ddot{V}_{cav} + \frac{1}{R_L C} \dot{V}_{cav} + \frac{1}{LC} V_{cav} = \frac{1}{C} \dot{I}_{cav}$$

$$\frac{1}{R_L C} = \frac{\omega_0}{Q_L}$$

$$\frac{1}{LC} = \omega_0^2$$

$$\ddot{V}_{cav} + \frac{\omega_0}{Q_L} \dot{V}_{cav} + \omega_0^2 V_{cav} = \frac{1}{C} \dot{I}_{cav}$$

$$V_{\text{hom}} = e^{-\frac{\omega_0 t}{2Q_L}} (C_1 e^{i\alpha t} + C_2 e^{-i\alpha t})$$

$$\alpha = \omega_0 \sqrt{1 - \frac{1}{4Q_L^2}}$$

One particular solution can be found with

$$I_{cav} = \hat{I} e^{i\omega t}$$

$$V_{cav} = \hat{V} e^{i(\omega t + \phi)}$$

Φ is the angle between the generator current and the resonator voltage

Derivation of Filling and Flattop Powers

$$V_{\text{par}} = \frac{R_L \hat{I} e^{i(\omega t + \phi)}}{\sqrt{1 + \tan^2 \phi}}$$

$$\text{with } \tan \phi = R \left(\frac{1}{\omega L} - \omega C \right) = Q \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)$$

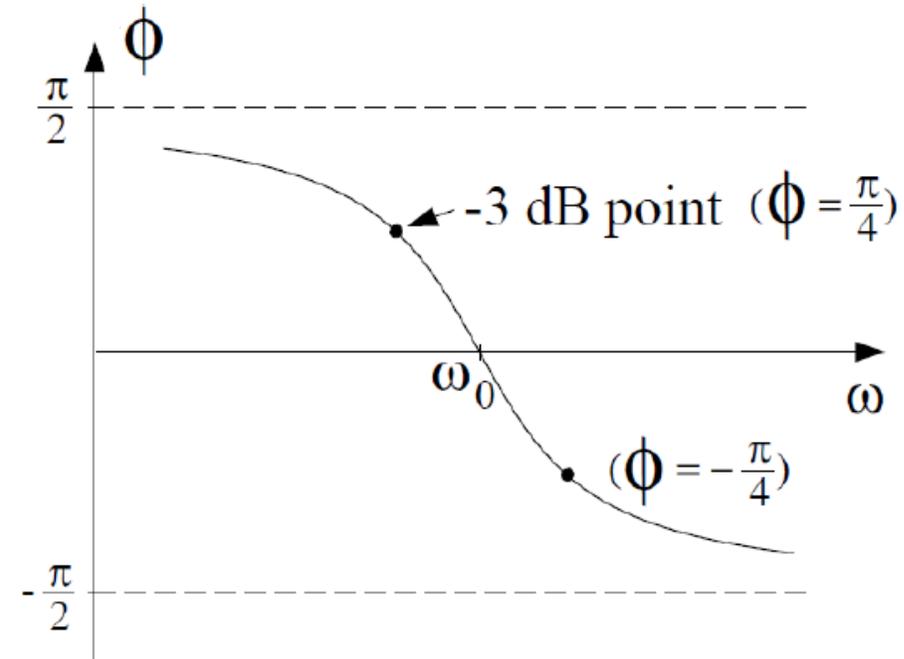
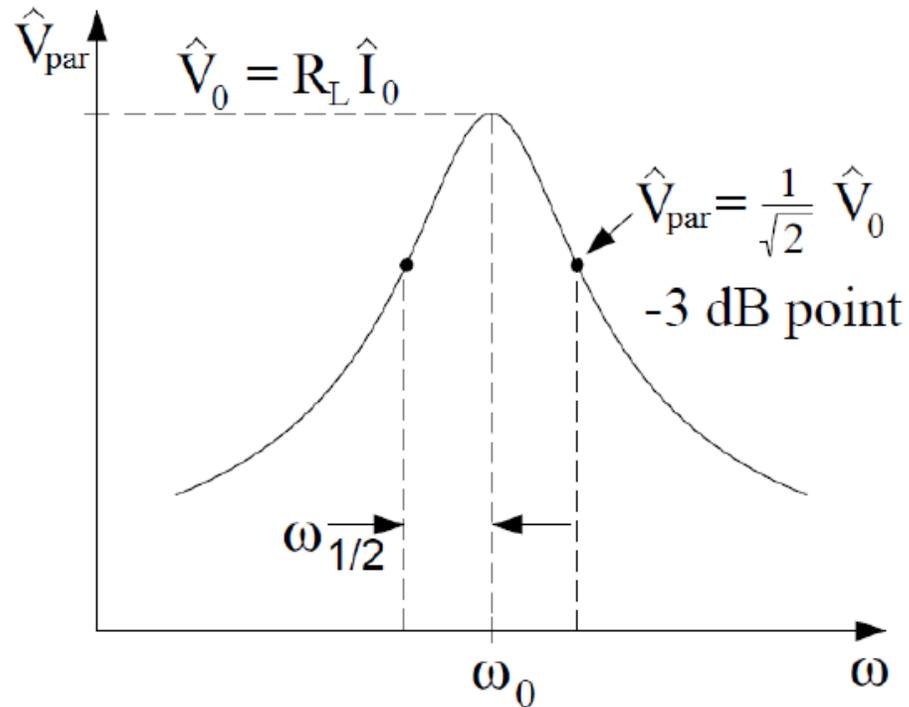
The particular solution is also called a stationary solution. If the generator frequency ω is very close to the resonance frequency ω_0 , the following approximation can be done:

$$\hat{V}_{\text{par}}(\Delta\omega) \approx \frac{R_L \hat{I}}{\sqrt{1 + \left(2Q_L \frac{\Delta\omega}{\omega}\right)^2}}$$

$$\text{where } \Delta\omega = \omega_0 - \omega$$

Derivation of Filling and Flattop Powers

The frequency dependency of the amplitude is known as the Lorentz curve



Bandwidth of the cavity is defined by the -3 dB point

Derivation of Filling and Flattop Powers

The general solution is:

$$V_{cav} = V_{hom} + V_{par} = e^{-\frac{\omega_0 t}{2Q_L}} (C_1 e^{i\alpha t} + C_2 e^{-i\alpha t}) + \frac{R_L \hat{I} e^{i(\omega t - \phi)}}{\sqrt{1 + \tan^2 \phi}}$$

since $Q_L \gg 1$ One can approximate: $\alpha \approx \omega_0$

for $C_1 = C_2 = -\frac{R_L \hat{I}}{2}$

$$V_{fill} = V_0 \left(1 - e^{-\frac{t}{\tau}}\right)$$

with $V_0 = R_L \hat{I} \approx 2R_L I_g = \frac{r}{Q} Q_L I_g$ $\tau = \frac{2Q_L}{\omega_0}$

\hat{I} represents an AC current

I_g represents a DC current

$$\hat{I} \approx 2I_g$$

$$\hat{I}_b \approx 2I_{b0}$$

$$I_{cav} = 2I_g - 2I_{b0}$$

$$V_{flat} = \frac{r}{Q} Q_L \left(I_g \left(1 - e^{-\frac{t}{\tau}}\right) - I_{b0} \cos(\phi_b) \left(1 - e^{-\frac{t - T_{inj}}{\tau}}\right) \right)$$

Derivation of Filling and Flattop Powers

$$\frac{dV_{\text{flat}}}{dt} = 0 \quad \text{We would like to have a constant voltage over the flattop}$$

$$\frac{d}{dt} \frac{r}{Q} Q_L \left(I_g \left(1 - e^{-\frac{t}{\tau}} \right) - I_{b0} \left(1 - e^{-\frac{t-T_{inj}}{\tau}} \right) \right) = 0$$

$$\frac{d}{dt} \frac{r}{Q} Q_L \left(I_g - I_g e^{-\frac{t}{\tau}} - I_{b0} + I_{b0} e^{-\frac{t-T_{inj}}{\tau}} \right) = 0$$

$$\frac{r}{Q} Q_L \left(I_g \frac{1}{\tau} e^{-\frac{t}{\tau}} - I_{b0} \frac{1}{\tau} e^{-\frac{t-T_{inj}}{\tau}} \right) = 0$$

$$I_g e^{-\frac{t}{\tau}} = I_{b0} e^{-\frac{t-T_{inj}}{\tau}}$$

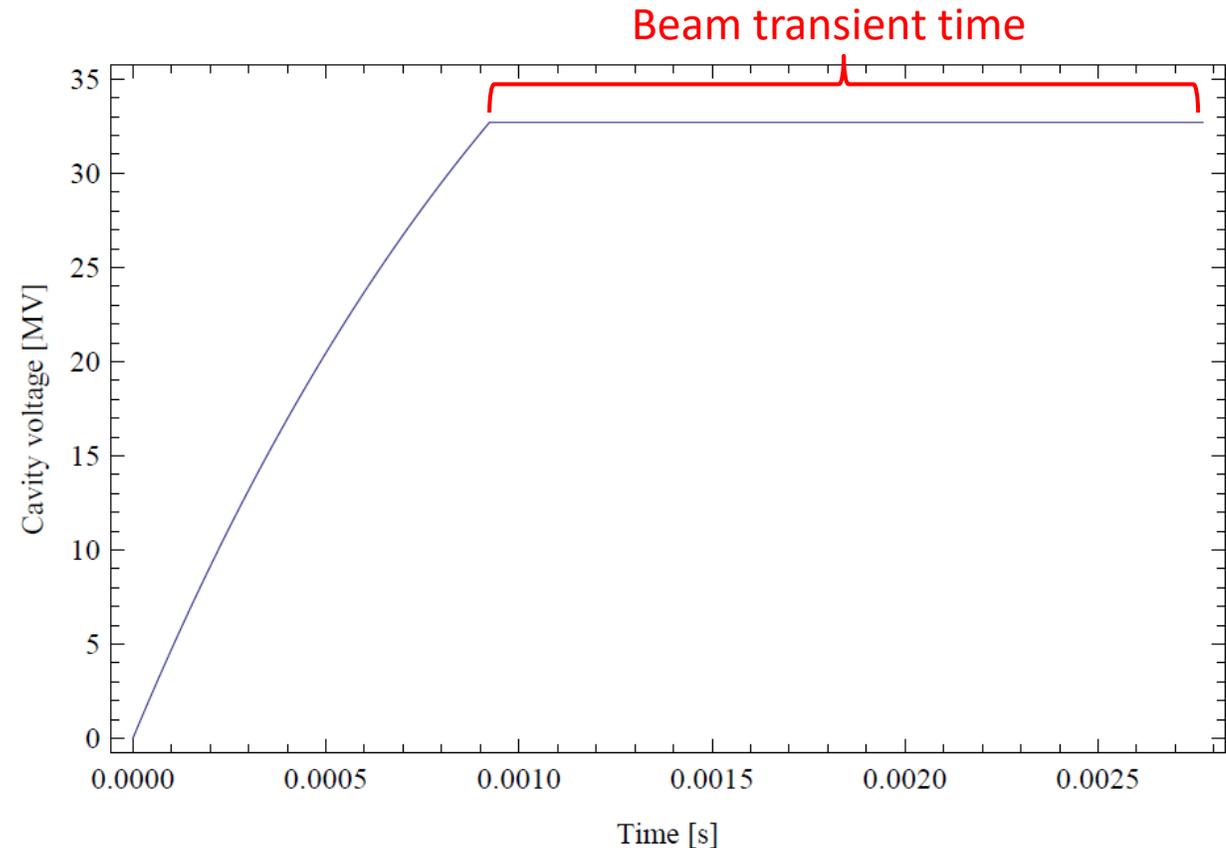
$$I_g = I_{b0} e^{\frac{T_{inj}}{\tau}}$$

Derivation of Filling and Flattop Powers

$$V_{\text{fill}} = V_0 \left(1 - e^{-\frac{t}{\tau}} \right)$$

$$V_{\text{flat}} = \frac{r}{Q} Q_L I_{b0} \left(e^{\frac{T_{\text{inj}} \omega_0}{2Q_L}} - 1 \right)$$

Parameter	Value
Filling time	923 μs
Beam current	5.8 mA
Q_L	5.44E6



Derivation of Filling and Flattop Powers

$$P = \frac{1}{4} \frac{r}{Q} Q_L I_g^2$$

$$P_{\text{fill}} = \frac{V_{\text{cav}}^2}{4 \frac{r}{Q} Q_L \left(1 - e^{-\frac{T_{\text{inj}} \omega_0}{2Q_L}}\right)^2}$$

$$V_{\text{cav}} = 31.5 \text{ MV/m} \cdot 1.038 \text{ m} = 32.7 \text{ MV}$$

$$Q_L = 5.44 \cdot 10^6$$

$$T_{\text{inj}} = 923 \text{ } \mu\text{s}$$

P_{fill} is 190 kW

$$P_{\text{flat}} = \frac{V_{\text{cav}}^2}{4 \frac{r}{Q} Q_L} \left(1 + \frac{\frac{r}{Q} Q_L I_{b0}}{V_{\text{cav}}}\right)^2$$

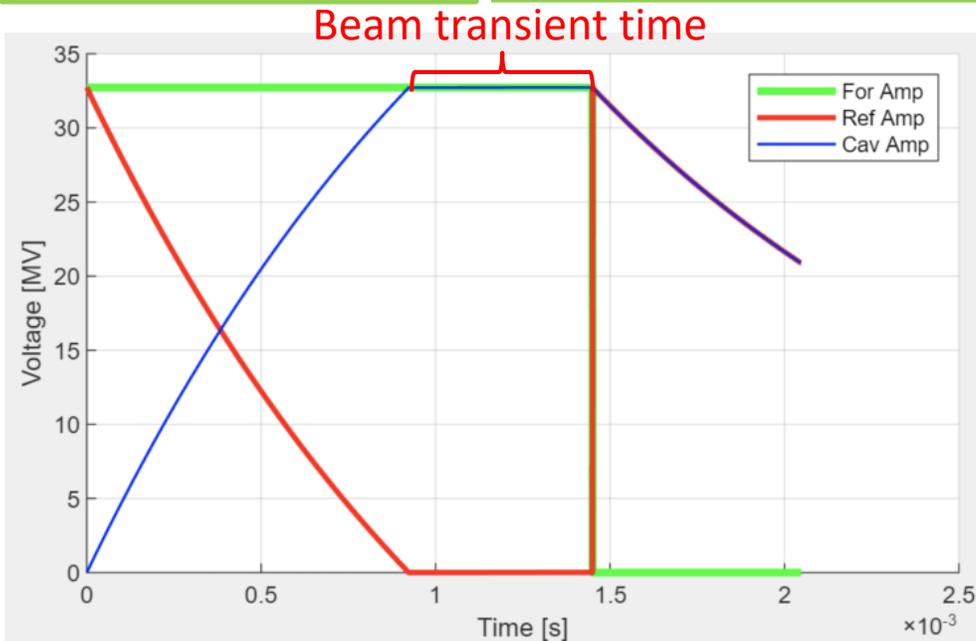
$$I_{b0} = 5.8 \text{ mA} \quad \phi_b = 180^\circ$$

P_{flat} is 190 kW

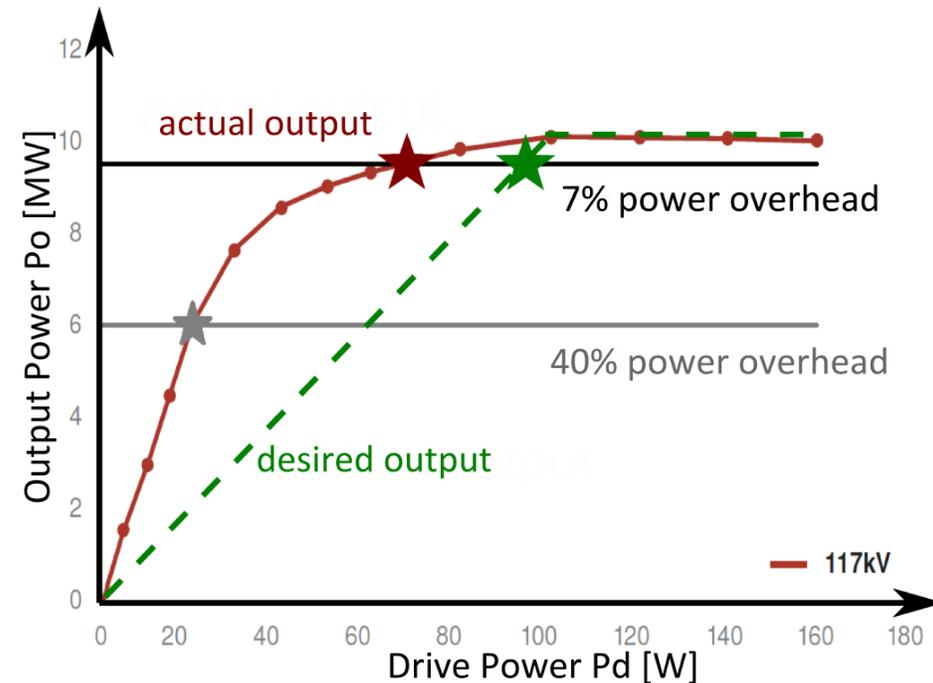
Derivation of Filling and Flattop Powers

- One can stay at a single working point of the power amplifier throughout the whole RF pulse

P_{fill} is 190 kW P_{flat} is 190 kW



Non-linear behavior of a klystron (red curve)



Derivation of Filling and Flattop Powers

- Set of equations for finding optimal parameters

The optimal coupling β_{opt}

$$\beta_{opt} = 1 + \frac{\frac{r}{Q} Q_0 I_{b0}}{V_{cav}} \cos(\phi_b)$$

Minimum power for maintaining the cavity voltage

$$P_{min} = \beta_{opt} \frac{V_{cav}^2}{\frac{r}{Q} Q_0}$$

Optimum tuning angle

$$\tan(\phi_{opt}) = -\frac{\frac{r}{Q} Q_{L,opt} I_{b0}}{V_{cav}} \sin(\phi_b)$$

For superconducting cavities one can simplify

$$Q_{L,opt} = \frac{V_{cav}}{\frac{r}{Q} I_{b0} \cos(\phi_b)}$$

$$\phi_{opt} = -\phi_b$$

$$P_{flat,min} = \frac{V_{cav}^2}{\frac{r}{Q} Q_{L,opt}} = V_{cav} I_{b0} \cos(\phi_b)$$

Example set of parameter

$$V_{cav} = 31.5 \text{ MV/m} \cdot 1.038 \text{ m} = 32.7 \text{ MV}$$

$$I_{b0} = 5.8 \text{ mA}$$

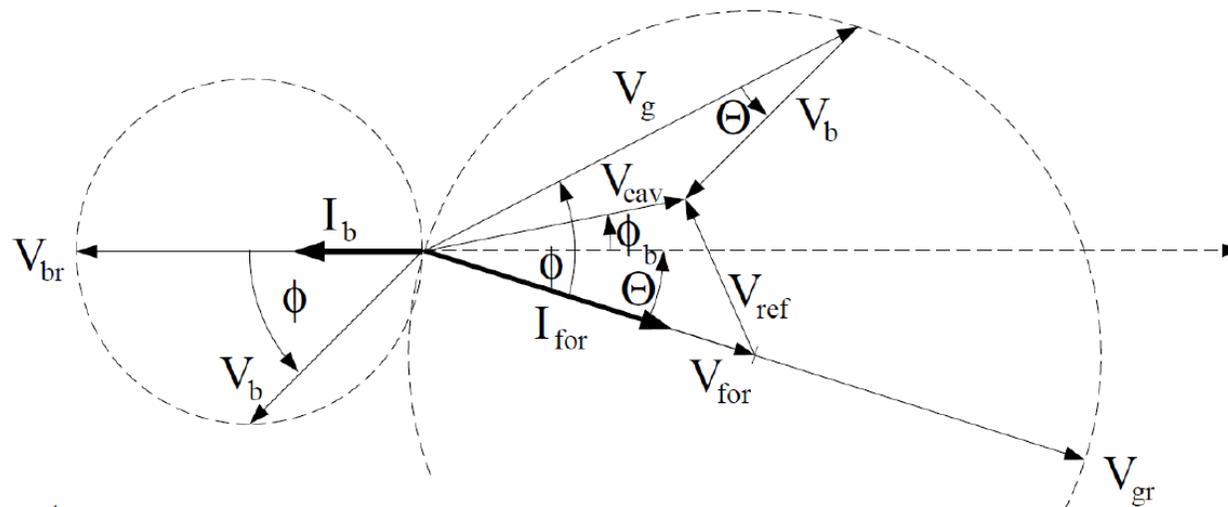
$$\cos(\phi_b) = 1$$

$$Q_{L,opt} = 5.44 \cdot 10^6$$

$$P_{flat,min} = 190 \text{ kW}$$

Detuned Cavity with Beam Loading

In reality cavities are detuned by the tuning angle Φ . The sources are Lorentz force detuning and microphonics.



$$V_{cav} = V_g + V_b$$

$$V_{cav} = V_{for} + V_{ref}$$

$$V_{cav} = \frac{1}{2} \frac{1}{1 + \tan^2(\phi)} (1 + i \tan(\phi)) \frac{r}{Q} Q_L (2I_g + 2I_{b0})$$

$$V_g = \frac{1}{\sqrt{1 + \tan^2(\phi)}} (1 + i \tan(\phi)) \frac{r}{Q} Q_L I_g = \cos(\phi) e^{i\phi} \frac{r}{Q} Q_L I_g$$

$$V_b = \frac{1}{2} \frac{1}{1 + \tan^2(\phi)} (1 + i \tan(\phi)) \frac{r}{Q} Q_L I_{b0} = \frac{1}{2} \cos(\phi) e^{i\phi} \frac{r}{Q} Q_L I_{b0}$$

Cavity Differential Equation Continues in Time

Differential equation for a driven LCR circuit

$$\ddot{\mathbf{V}}(t) + \frac{\omega_0}{Q_L} \dot{\mathbf{V}}(t) + \omega_0^2 \mathbf{V}(t) = \frac{\omega_0 R_L}{Q_L} \dot{\mathbf{I}}(t)$$

$$\frac{\omega_0}{Q_L} \ll \omega_0 \quad \text{The cavity is a weakly damped system}$$

$$\omega_{res} = \omega_0 \quad \text{good approximation, since } \omega_{res} = \omega_0 \sqrt{1 - \frac{1}{4Q_L^2}} \approx \omega_0$$

Driving current I_g and Fourier component I_b of pulsed beam are harmonic with time dependence $e^{i\omega t}$.
Therefore, we separate the fast RF oscillation from the real and imaginary parts of the field vector.

$$\mathbf{V}(t) = (V_r(t) + iV_i(t)) \cdot e^{i\omega t}$$

$$\mathbf{I}(t) = (I_r(t) + iI_i(t)) \cdot e^{i\omega t}$$

Insertion in equation above and omission of the second-order time derivatives of V yields...

Cavity Differential Equation Continues in Time

... the first-order differential equation for the envelope:

$$\dot{V}_r + \omega_{1/2} V_r + \Delta\omega V_i = R_L \omega_{1/2} I_r$$

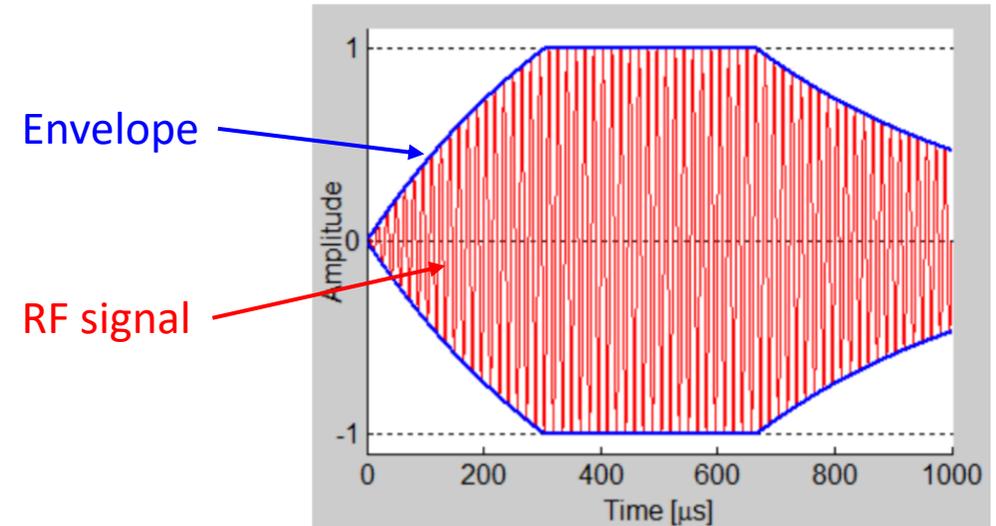
$$\dot{V}_i + \omega_{1/2} V_i - \Delta\omega V_r = R_L \omega_{1/2} I_i$$

with $\omega_{1/2} = \frac{\omega_0}{2Q_L}$ cavity bandwidth
 $\Delta\omega = \omega_0 - \omega$ cavity detuning

In state space formalism

$$\frac{d}{dt} \begin{pmatrix} V_r \\ V_i \end{pmatrix} = \begin{pmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} V_r \\ V_i \end{pmatrix} + \begin{pmatrix} R_L \omega_{1/2} & 0 \\ 0 & R_L \omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} I_r \\ I_i \end{pmatrix}$$

$$\dot{x}(t) = \mathbf{A} \cdot x(t) + \mathbf{B} \cdot u(t)$$



$$\mathbf{A} = \begin{pmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} R_L \omega_{1/2} & 0 \\ 0 & R_L \omega_{1/2} \end{pmatrix}$$

$$x = \begin{pmatrix} V_r \\ V_i \end{pmatrix}$$

$$u = \begin{pmatrix} I_r \\ I_i \end{pmatrix}$$

Cavity Differential Equation Continuous and Discrete in Time

$$\frac{d}{dt} \begin{pmatrix} V_r \\ V_i \end{pmatrix} = \begin{pmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} V_r \\ V_i \end{pmatrix} + \begin{pmatrix} R_L\omega_{1/2} & 0 \\ 0 & R_L\omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} I_r \\ I_i \end{pmatrix}$$

$$\begin{bmatrix} V_{i,n} \\ V_{q,n} \end{bmatrix} = \begin{bmatrix} 1 - T\omega_{1/2} & -T\Delta\omega \\ T\Delta\omega & 1 - T\omega_{1/2} \end{bmatrix} \begin{bmatrix} V_{i,n-1} \\ V_{q,n-1} \end{bmatrix} + T\omega_{1/2}R_L \begin{bmatrix} I_{i,n-1} \\ I_{q,n-1} \end{bmatrix}$$

Cavity Simulator Live Demo

- Demo of single cavity in pulsed operation
 - E.g., let's check the parameter set we have derived earlier

$$V_{cav} = 31.5 \text{ MV/m} \cdot 1.038 \text{ m} = 32.7 \text{ MV}$$

$$I_{b0} = 5.8 \text{ mA}$$

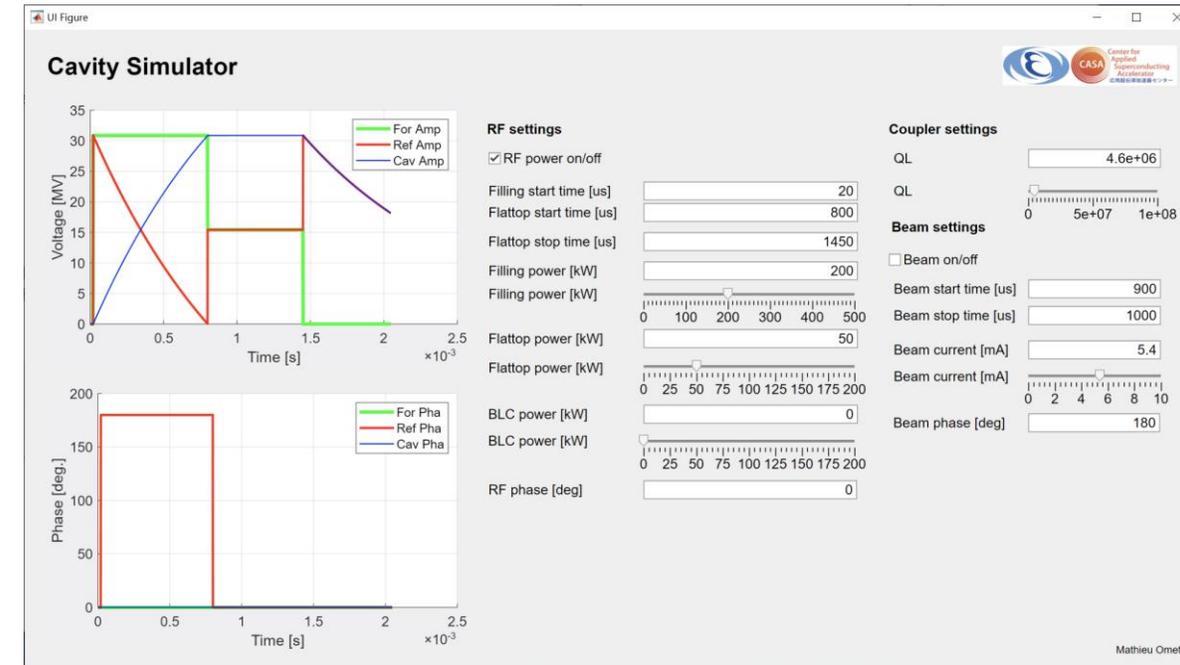
$$\cos(\phi_b) = 1$$

$$Q_{L,opt} = 5.44 \cdot 10^6$$

$$P_{flat,min} = 190 \text{ kW}$$

$$T_{inj} = 923 \text{ } \mu\text{s}$$

- Let's see for what kind of operation low and high Q_L values are interesting



LLRF Systems

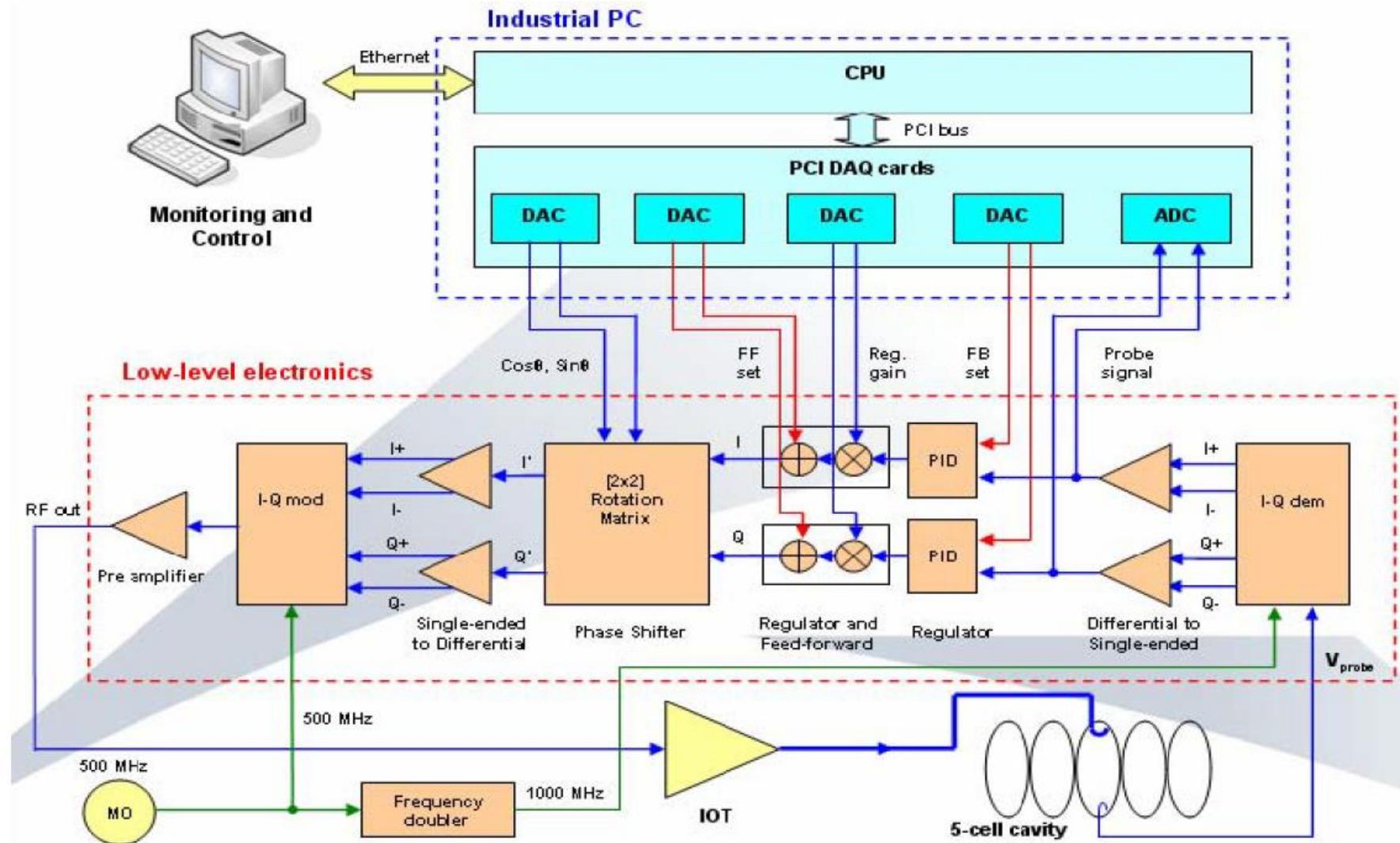
Types of LLRF Systems



- Analog
 - Designed, optimized and built for a specific purpose
 - Hard to modify
 - Need extra hardware for e.g. data recording

- Digital
 - More flexibility
 - On how to design the system
 - Always possible to add, change, tweak digital algorithms
 - Modern algorithms can be realized
 - Remotely maintainable to a large degree

Example of an Analog LLRF System

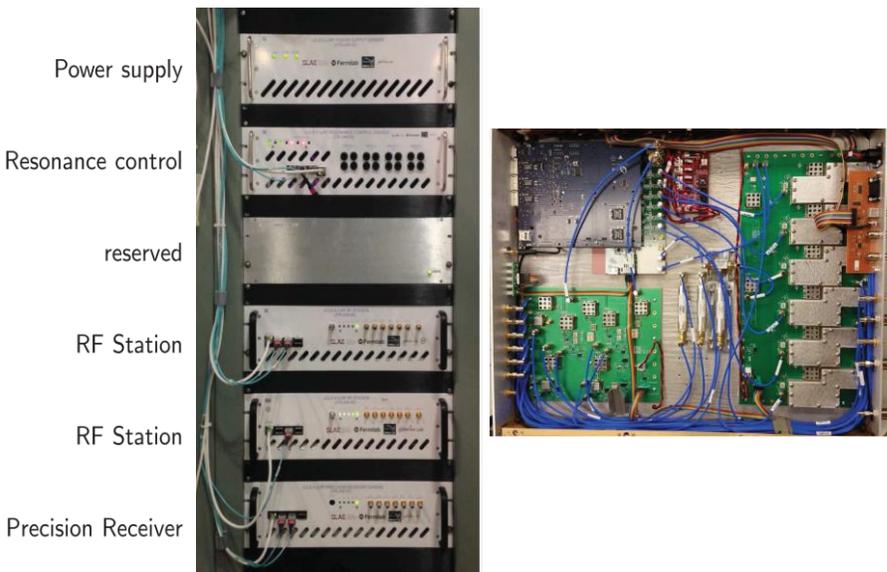


Types of Digital LLRF Systems

- 19-inch modules (“Pizza box”)
 - Individually developed and built hardware
 - Well optimized

- Crate-based systems
 - Of-the-shelf components
 - Well optimized cards available
 - Highly modular

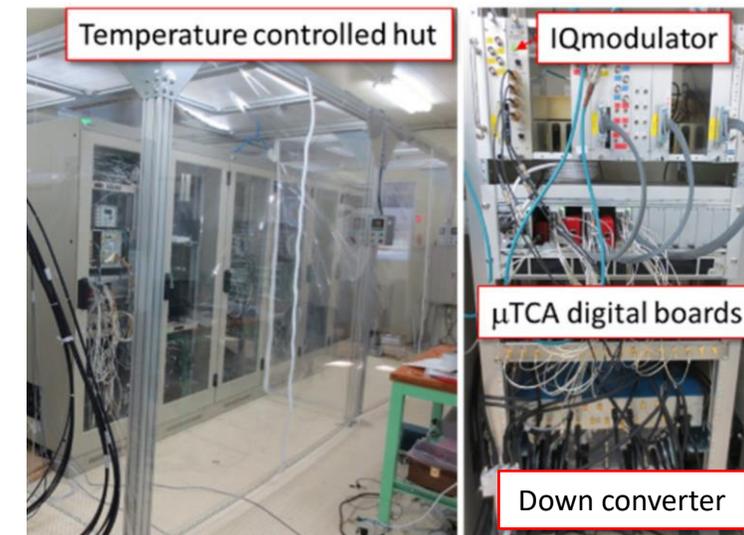
- Mixed systems
 - Best of both worlds



LLCLS-II prototype LLRF system at FNAL CMTS

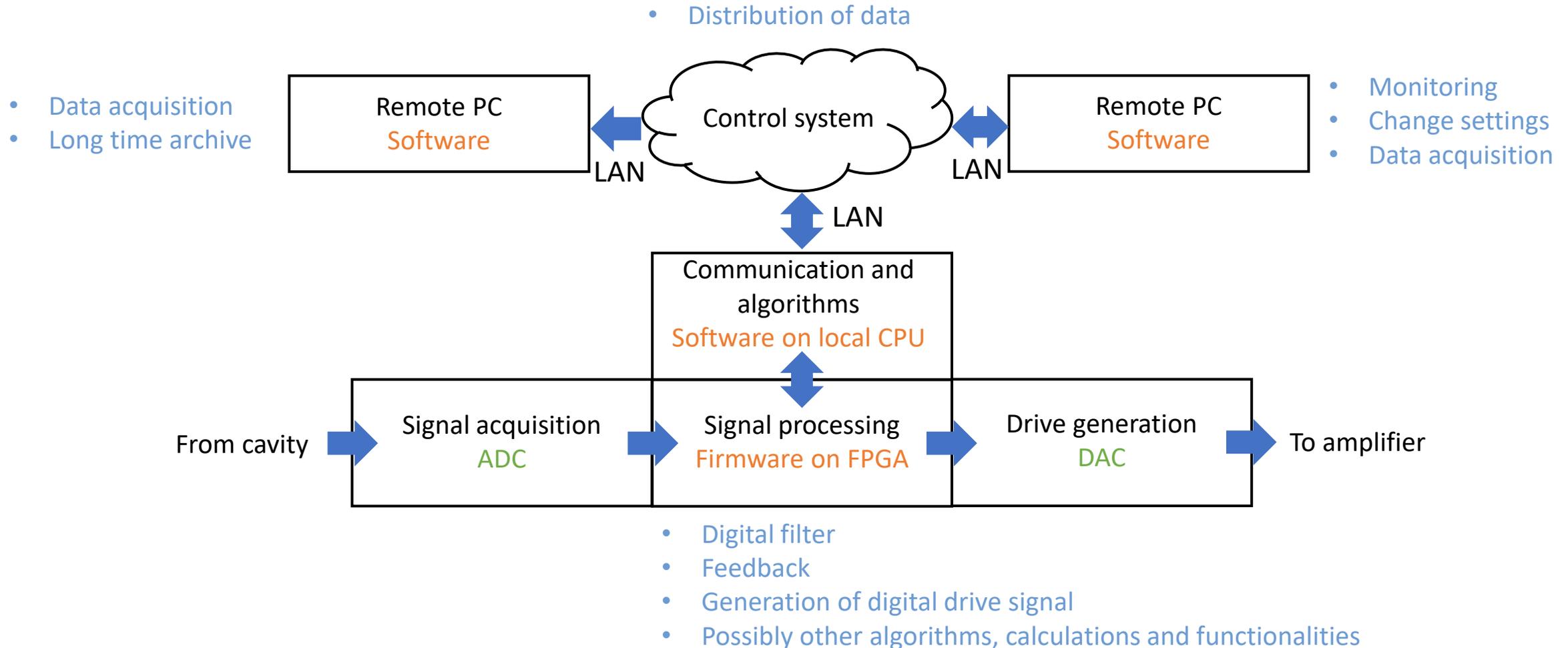


μTCA.4-based LLRF systems at European XFEL at DESY



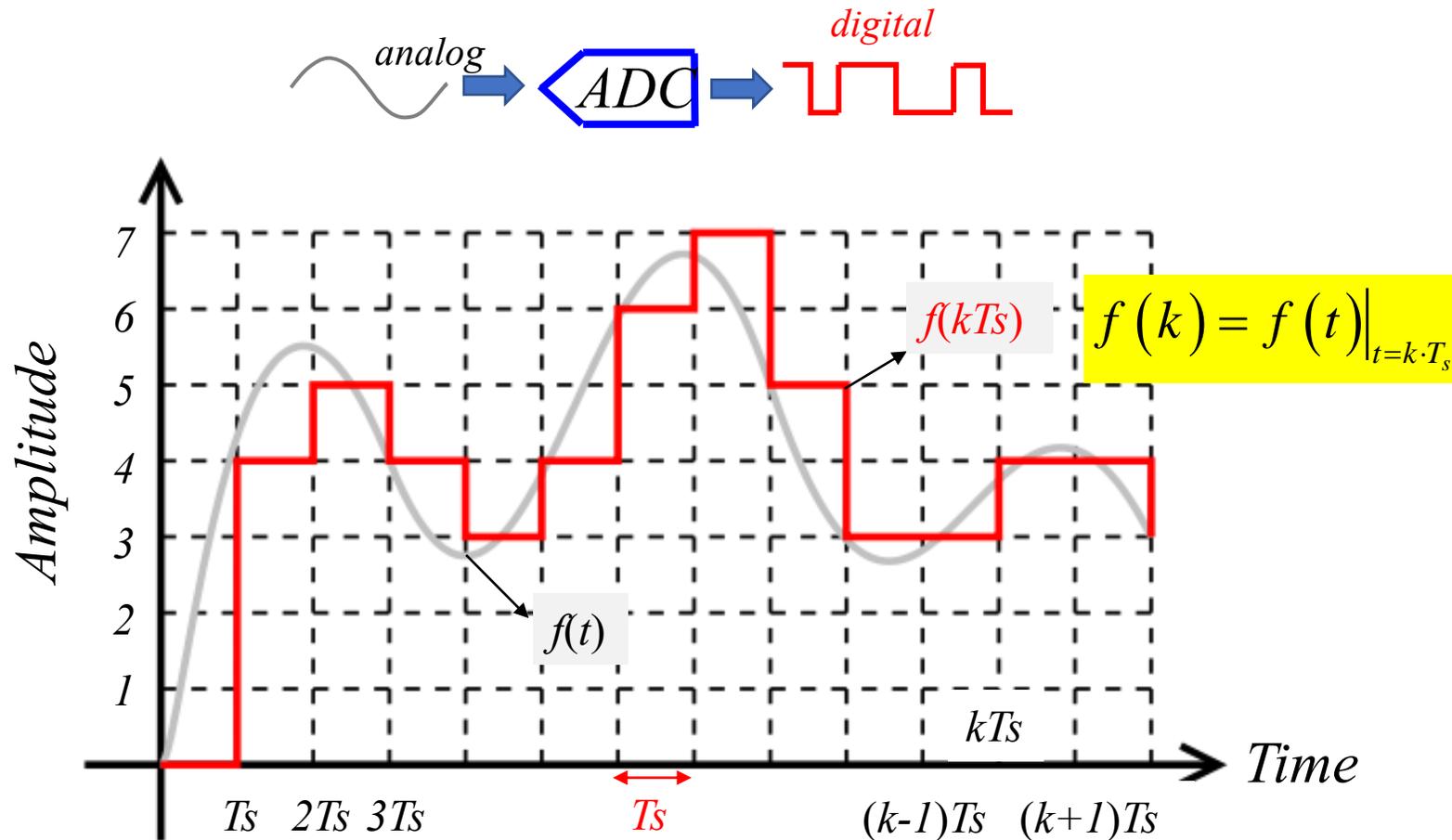
μTCA.0-based LLRF system at cERL at KEK

System Architecture Example

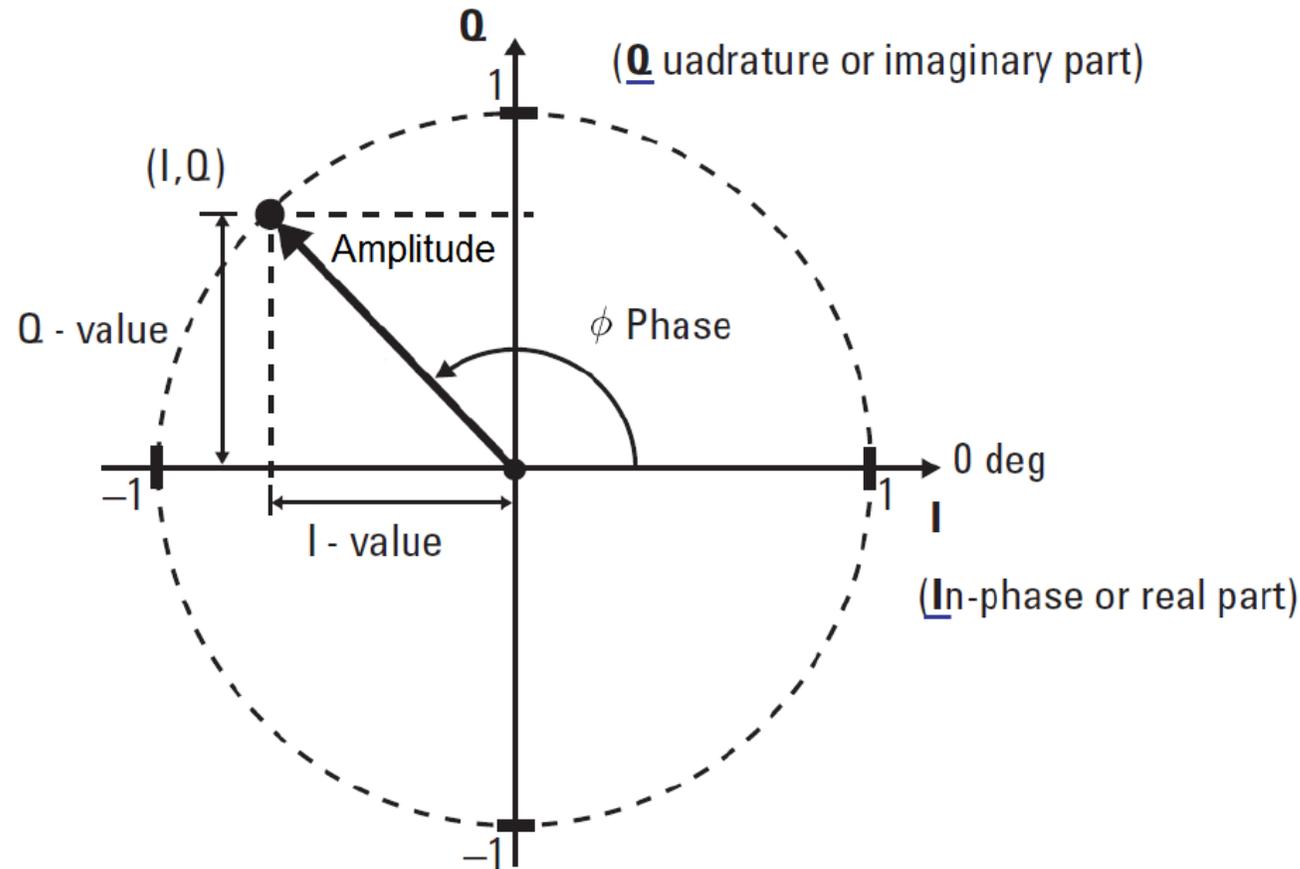


Signal Sampling

Goal: Convert an Analog Signal into a Digital Signal



Representation in Quadrature and In-phase



$$I = A \cos(\phi)$$

$$Q = A \sin(\phi)$$

$$A = \sqrt{I^2 + Q^2}$$

$$\phi = \text{atan} \left(\frac{Q}{I} \right)$$

Down Conversion in Frequency

- Nyquist-Shannon theorem: $f_s > 2f_{RF}$
 - If this is fulfilled, a perfect reconstruction of f_{RF} is guaranteed.

$$S_{RF}(t) = A_{RF} \cdot \sin(2\pi \cdot f_{RF} \cdot t + \phi_{RF})$$

$$\phi_{RF} = 0$$

$$S_{LO}(t) = A_{LO} \cdot \sin(2\pi \cdot f_{LO} \cdot t + \phi_{LO})$$

$$A_{LO} = 1 \quad \phi_{LO} = 0$$

$$S_{LO \cdot RF}(t) = \sin(2\pi \cdot f_{LO} \cdot t) \cdot \sin(2\pi \cdot f_{RF} \cdot t)$$

$$= \frac{1}{2} (\cos(2\pi \cdot (f_{LO} - f_{RF}) \cdot t) - \cos(2\pi \cdot (f_{LO} + f_{RF}) \cdot t))$$

$$S_{IF}(t) = \frac{1}{2} \cos(2\pi \cdot f_{IF} \cdot t)$$

Example frequencies

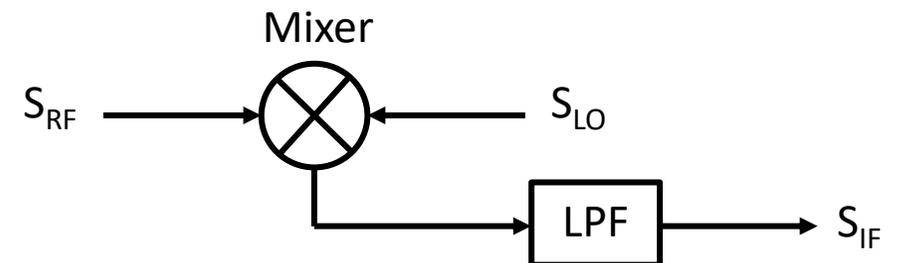
$$f_{RF} = 1.3 \text{ GHz}$$

$$f_{LO} = 1.31 \text{ GHz}$$

RF: Radio frequency
LO: Local oscillator
IF: Intermediate frequency

$$f_{IF} = 10 \text{ MHz}$$

- Preserves amplitude and phase information



Sampling methods



- IQ Sampling
- Under sampling & Over sampling

IQ Sampling

$$f_s = 4 \cdot f_{IF}$$

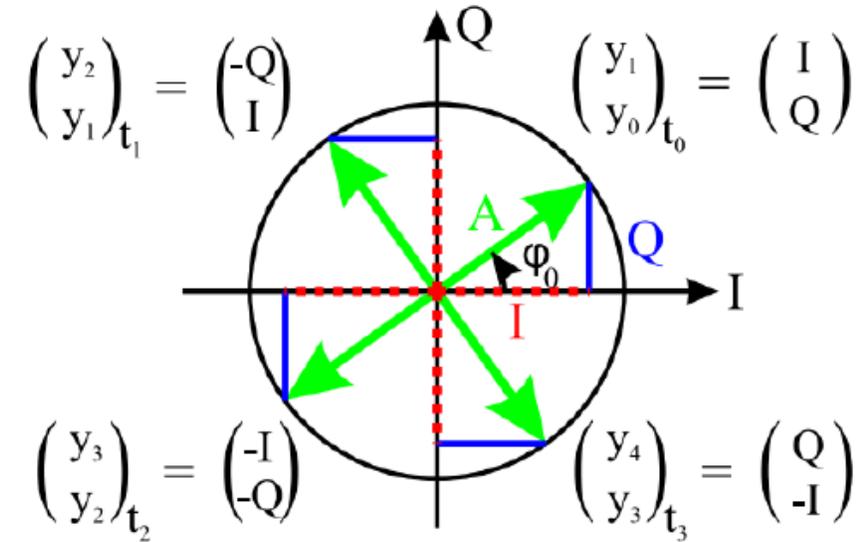
$$f_{IF}(0) = Q$$

$$f_{IF}\left(\frac{\pi}{2}\right) = I$$

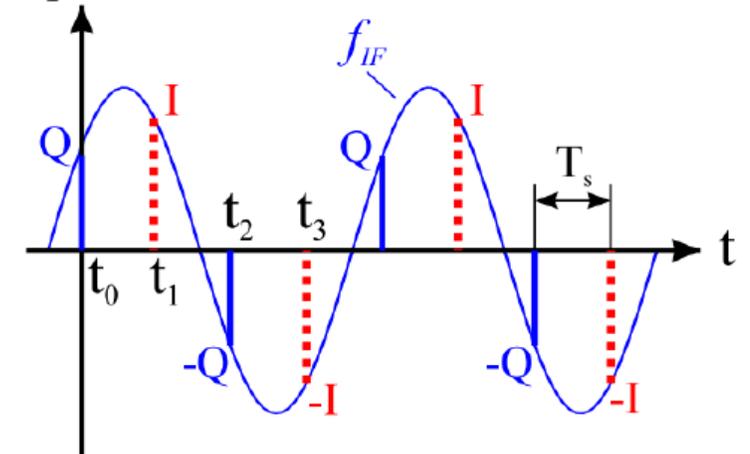
$$f_{IF}(\pi) = -Q$$

$$f_{IF}\left(\frac{3\pi}{2}\right) = -I$$

$$\begin{pmatrix} I \\ Q \end{pmatrix}_n = \begin{pmatrix} \cos(\Delta\phi_n) & -\sin(\Delta\phi_n) \\ \sin(\Delta\phi_n) & \cos(\Delta\phi_n) \end{pmatrix} \cdot \begin{pmatrix} f_{IF,n+1} \\ f_{IF,n} \end{pmatrix}$$



amplitude



Undersampling and Oversampling

$$\frac{f_s}{f_{IF}} = \frac{M}{L} = m$$

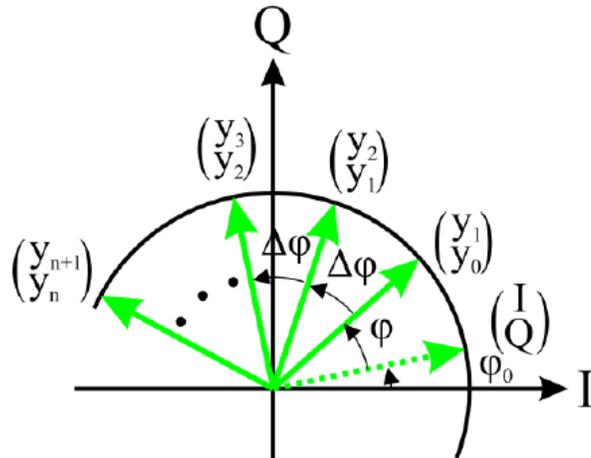
$$\Delta\phi = \frac{2\pi}{m}$$

$m = 4$ corresponds to the IQ sampling

$m < 2$ corresponds to undersampling

$m > 2$ to oversampling

$$\begin{pmatrix} I \\ Q \end{pmatrix}_n = \frac{1}{\sin(\Delta\phi + \phi)} \begin{pmatrix} \cos(n\Delta\phi + \phi) & -\cos((n+1)\Delta\phi + \phi) \\ -\sin(n\Delta\phi + \phi) & \sin((n+1)\Delta\phi + \phi) \end{pmatrix} \cdot \begin{pmatrix} y_{IF,n+1} \\ y_{IF,n} \end{pmatrix}$$



$$I = \frac{2}{m} \sum_{n=0}^{m-1} y_n \cos\left(\frac{2\pi n}{m}\right)$$

$$Q = \frac{2}{m} \sum_{n=0}^{m-1} y_n \sin\left(\frac{2\pi n}{m}\right)$$

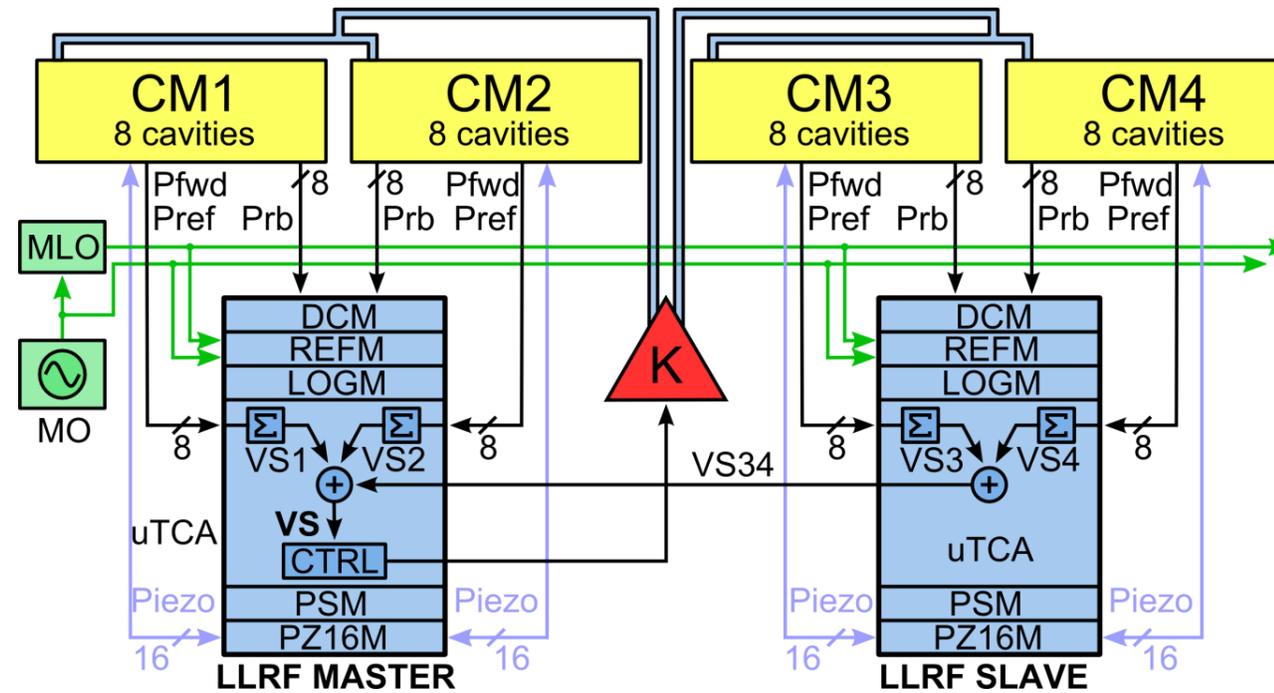
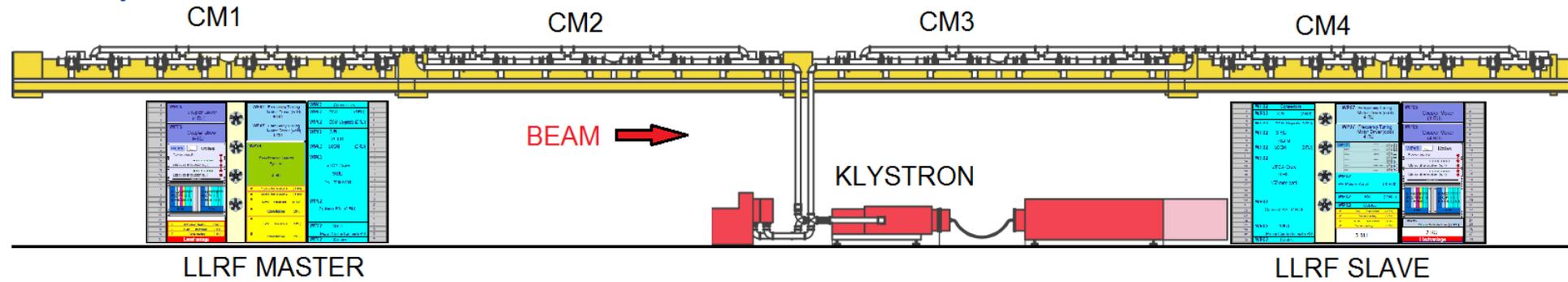
Undersampling and Oversampling



- Advantages of undersampling
 - Relaxed requirements for ADC due to lower sampling rate (possible cost reduction)
 - Relaxed requirements for FPGA due to lower data rate (possible cost reduction)
 - Possible to detect IF signals with higher frequency than the ADC sampling rate
- Advantages of oversampling
 - More sample points per period
 - Noise reduction due to averaging in the calculation of I and Q values
 - Choice of IF location in the first Nyquist zone is more flexible (corresponding to e.g. an available analog anti-aliasing low pass filter or to the ADC circuit optimization)

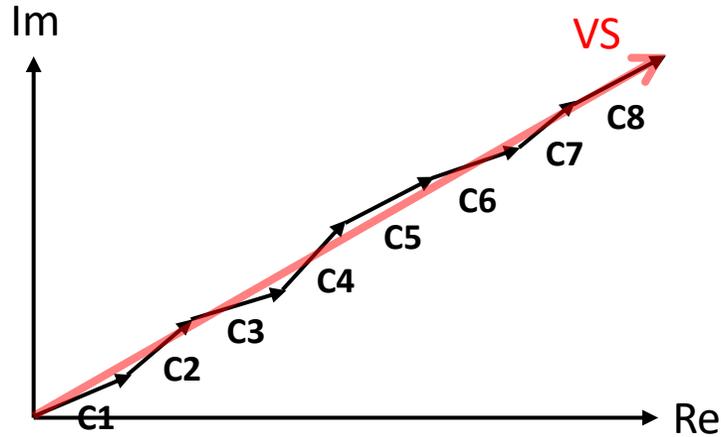
Digital Signal Processing and Implementation

Vector Sum Control of 32 Cavities at the European XFEL

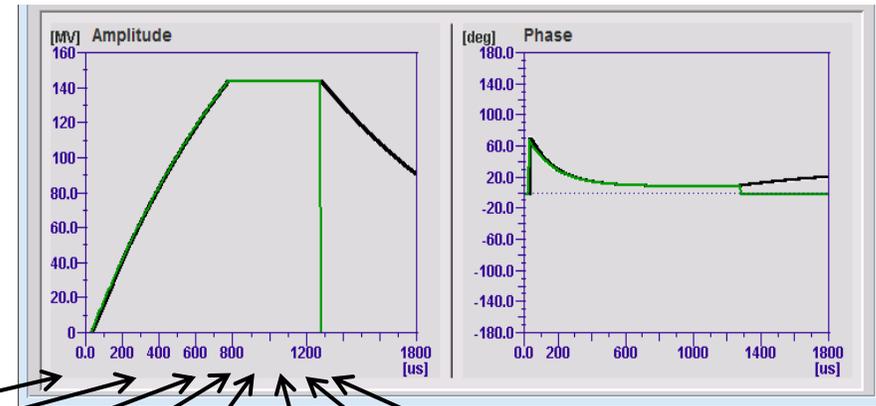


Vector Sum Control

- Drive multiple cavities with one power source



Vector Sum



Read back value
 Set Point



C1

C2

C3

C4

C5

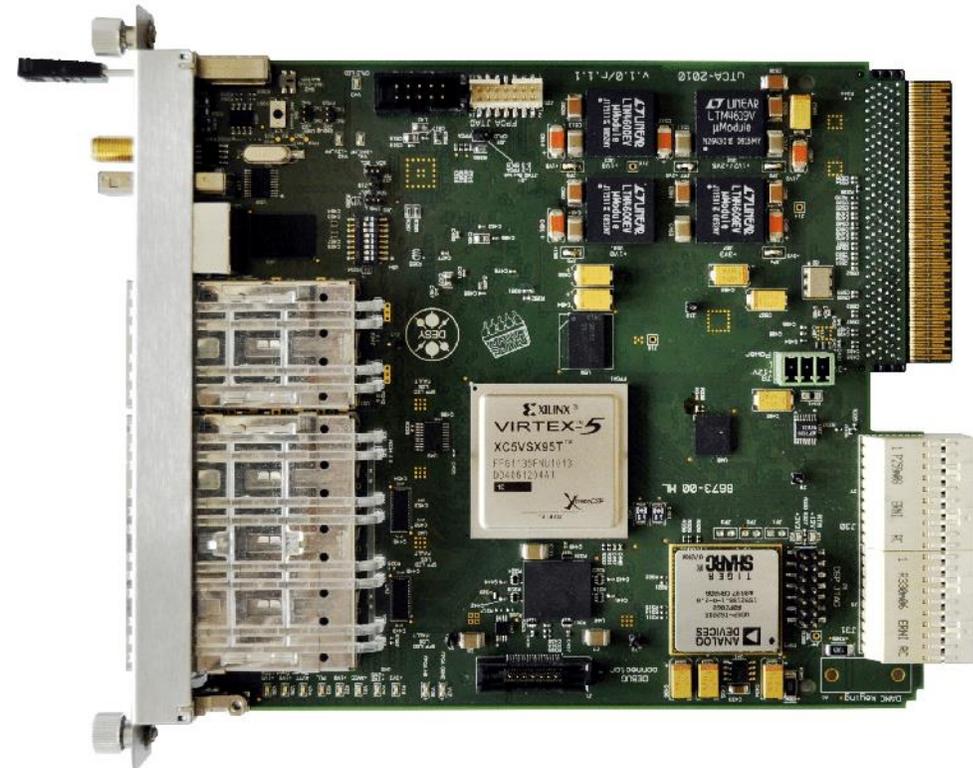
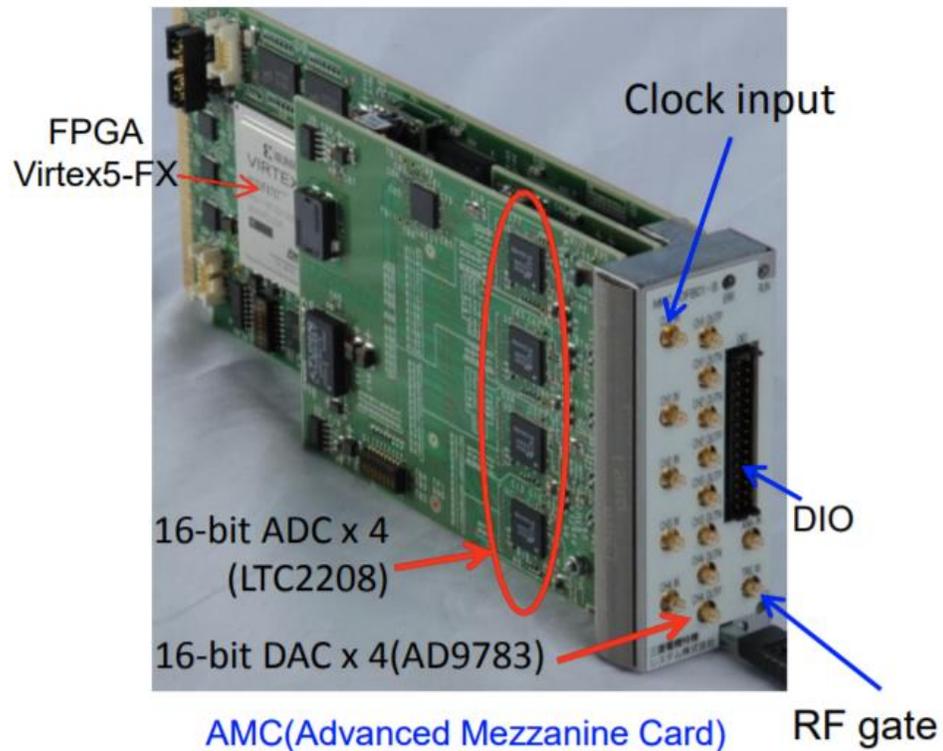
C6

C7

C8

Field-Programmable Gate Array (FPGA)

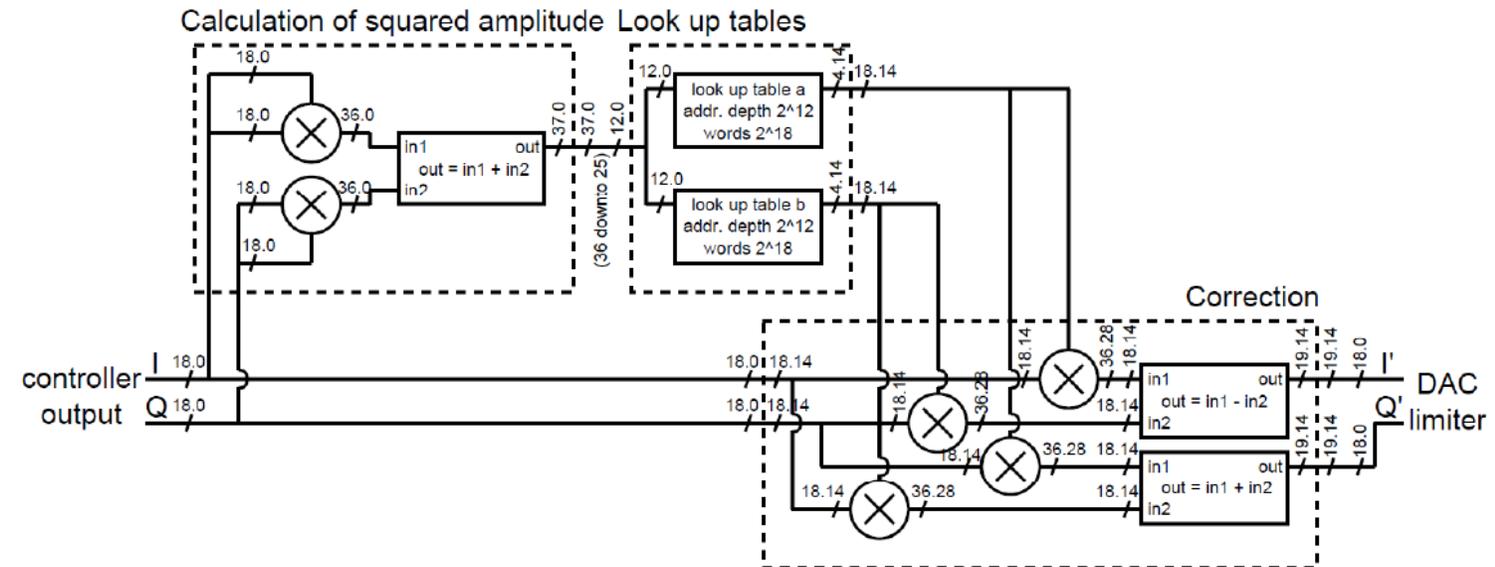
- Typically all time-critical digital signal processing is implemented on a FPGA



How to implement algorithms on a FPGA

- Write down the requirements for the firmware
- Make a flow chart and check signal widths
- Create your code
- Create a test bench for your code
- Test and debug your code within the test bench
- Test and debug your code on the target hardware (typically a test setup identical to the production system)
- Deploy the firmware on the production hardware
- If the requirements have changed, revise them and go through all previous steps

Example of Flowchart for a VHDL Algorithm for the FPGA



Ways to Create VHDL Code

- Write directly VHDL source code
 - Absolute control over functionality
 - Allows optimization for different goals (e.g. clock cycles, resources, etc.)
 - Needs good understanding
 - Can take longer to get to the result

```
-- (this is a VHDL comment)

-- import std_logic from the IEEE library
library IEEE;
use IEEE.std_logic_1164.all;

-- this is the entity
entity name_of_entity is
  port (
    IN1 : in std_logic;
    IN2 : in std_logic;
    OUT1: out std_logic);
end entity name_of_entity;

-- here comes the architecture
architecture name_of_architecture of name_of_entity is

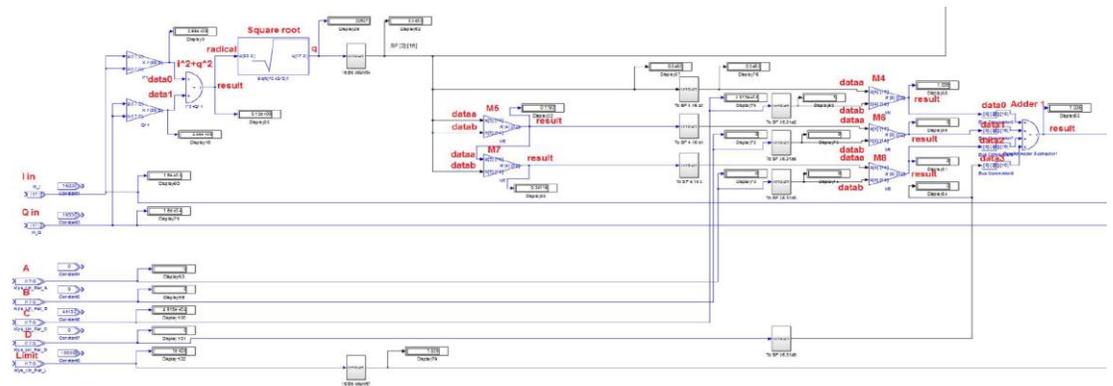
  -- Internal signals and components would be defined here

begin

  OUT1 <= IN1 and IN2;

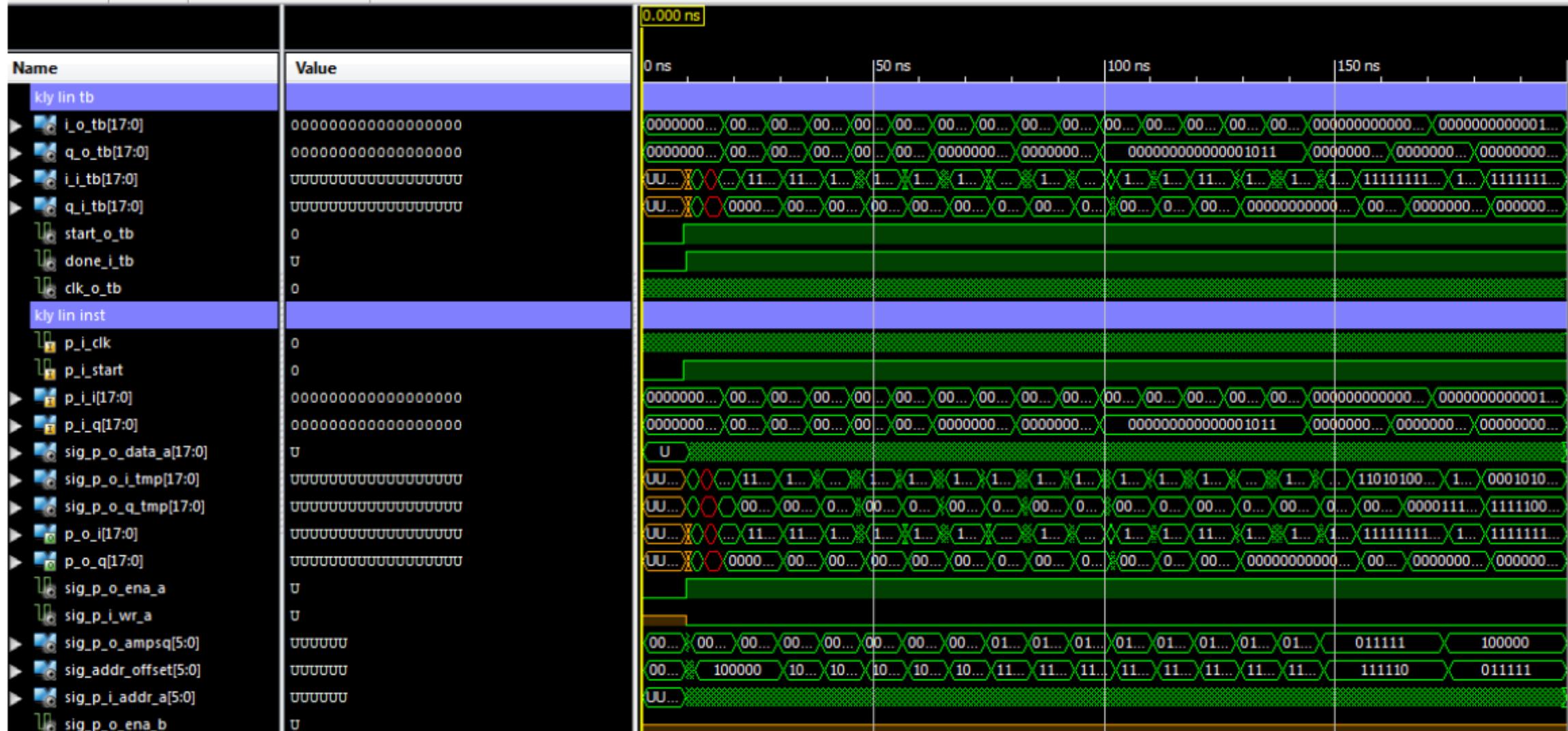
end architecture name_of_architecture;
```

- Use e.g. MathWorks Simulink to create VHDL code
 - Allows quick prototyping
 - Good graphical representation of signal flow
 - Less control
 - Creates VHDL code, which most times cannot be easily debugged by a human



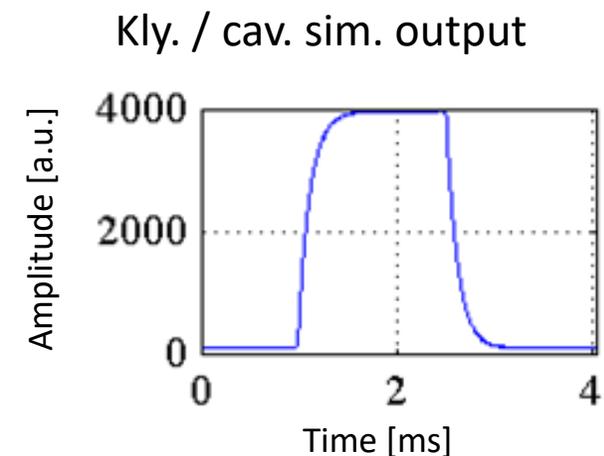
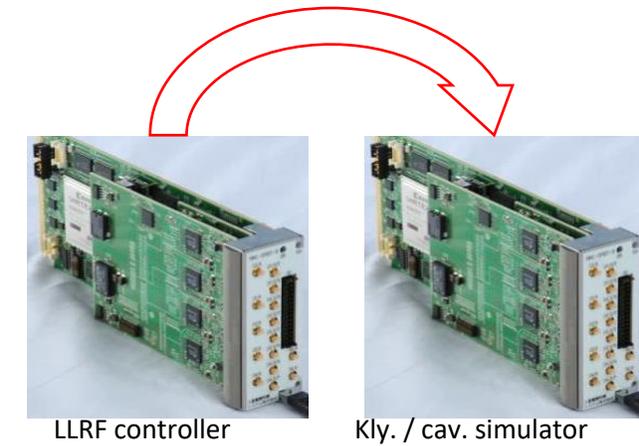
VHDL = VHSIC Hardware Description Language
VHSIC = Very High Speed Integrated Circuit

Test the Code on a Test Bench



FPGA-based Simulator

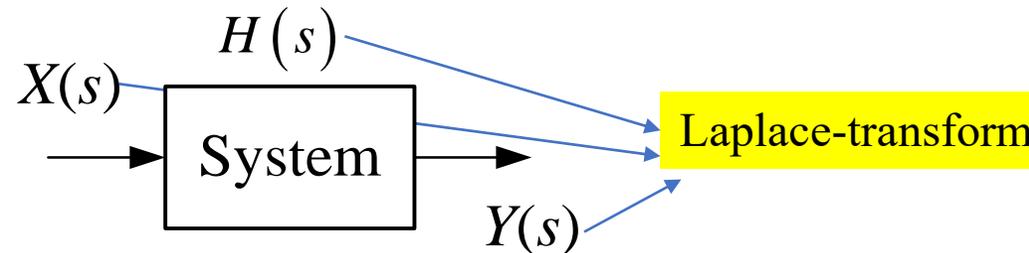
- Operating cavities is expensive (e.g, cryo in case of SRF cavities, high power amplifiers, etc.)
- Development and test also possibly with simulators
- Example implemented at KEK
 - Klystron simulator based on two direct lookup tables
 - Cavity simulator based on the time discrete cavity differential equation
 - Test setup realized in a development rack
 - Two μ TCA cards (Xilinx Virtex 5 FX)
 - LLRF controller
 - klystron / cavity simulator
 - Feedforward (open loop) and feedback (closed loop) operation modes worked



Controller Theory

Transfer Function

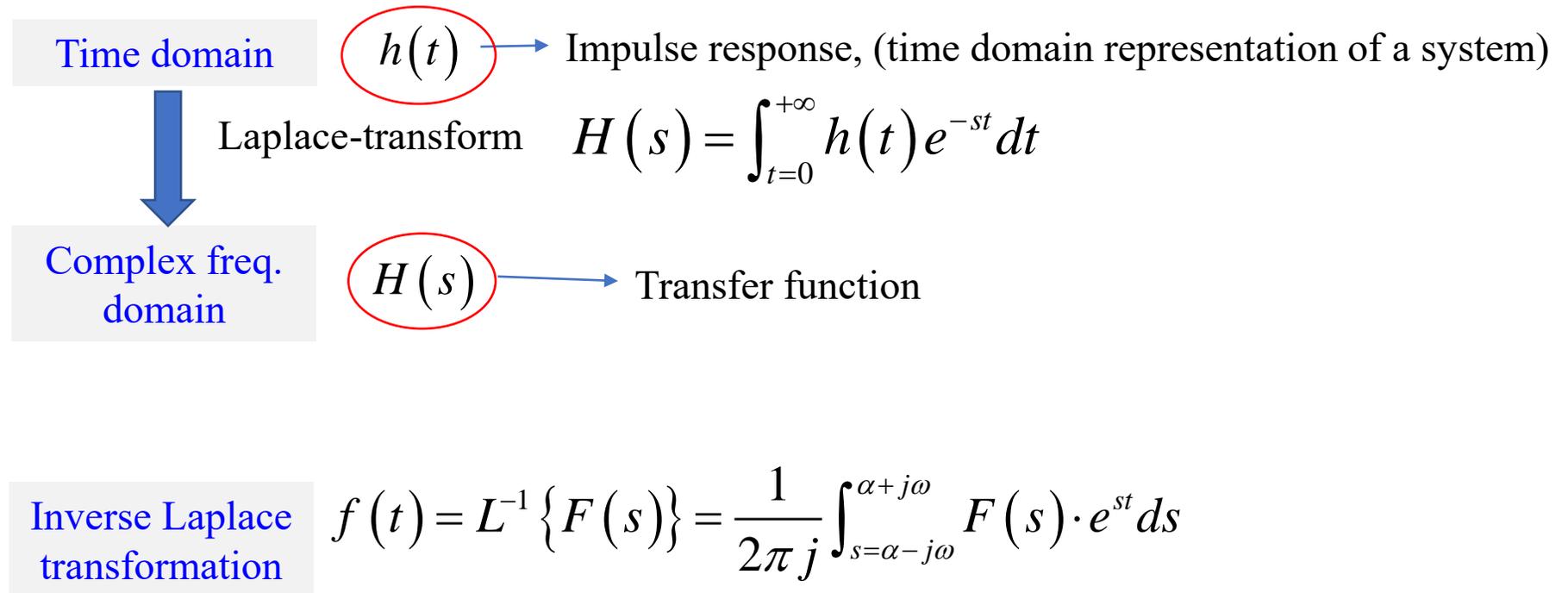
- A **Transfer Function** is the ratio of the output of a system to the input of a LTI system.
- The $X(s)$ and $Y(s)$ are the Laplace-transform of the input/output signal, respectively.
- Key point: The transfer function $H(s)$ includes information of a system (usually can be seen as a **representation of a given system**). i.e. if we know the transfer function $H(s)$ of a specified system (assume initial states = 0), we can calculate the output $Y(s)$ by input $X(s)$.



$$H(s) = \frac{Y(s)}{X(s)}, \quad Y(s) = H(s) \cdot X(s).$$

Laplace-Transform

- Formula of the Laplace-Transform



Laplace-Transform

TABLE 15.1

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as} F(s)$
Frequency shift	$e^{-at} f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2 f}{dt^2}$	$s^2 F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3 f}{dt^3}$	$s^3 F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \dots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(x) dx$	$\frac{1}{s} F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds} F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s) ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1 - e^{-sT}}$
Initial value	$f(0)$	$\lim_{s \rightarrow \infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s \rightarrow 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

TABLE 15.2

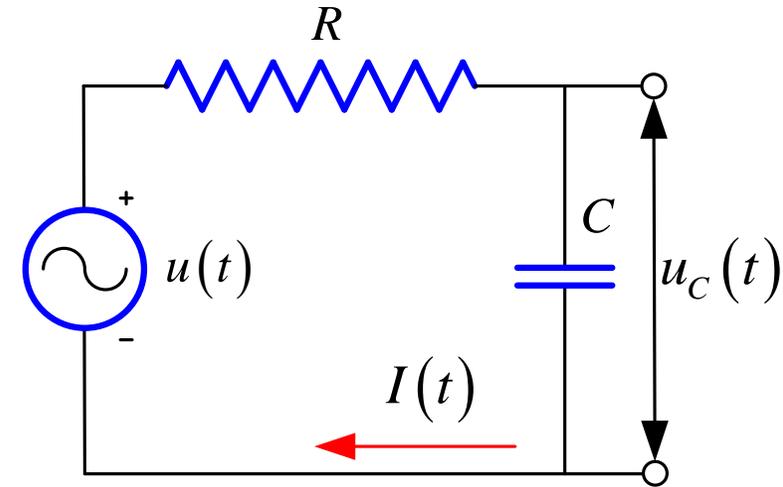
Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t + \theta)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
$\cos(\omega t + \theta)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$

*Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.

Hot to obtain a Transfer Function (Example: RC circuit)

- TF is related to the differential equation
- The differential equation reads:



$$I(t) = C \cdot \frac{du_c(t)}{dt} = \frac{u(t) - u_c(t)}{R}$$

Current on C

Current on R

then, $\frac{du_c(t)}{dt} + \frac{u_c(t)}{RC} = \frac{u(t)}{RC}, u_c(0^-) = 0$

Laplace Transform

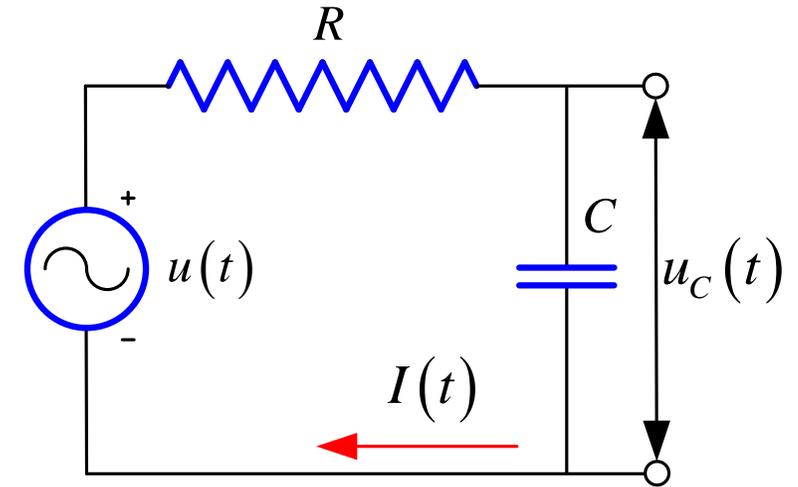
$$f'(t) \Leftrightarrow sF(s)$$

then, $sU_c(s) + \frac{U_c(s)}{RC} = \frac{U(s)}{RC}$ and $\frac{U_c(s)}{U(s)} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$

Transfer function

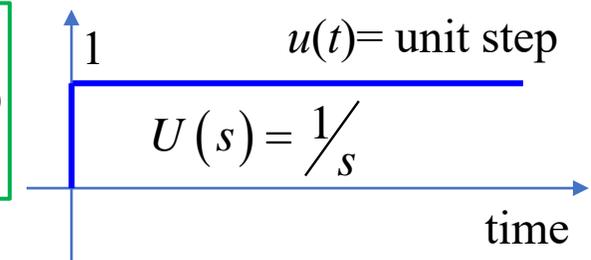
Example: RC circuit

- If we know the TF (assume initial state = 0), in principle, we know the system output $u_c(t)$ according to the given input $u(t)$ (unit step).



$$H(s) = \frac{U_c(s)}{U(s)} = \frac{1}{RC} \frac{1}{s + \frac{1}{RC}}$$

$$U_c(s) = \frac{1}{RC} \frac{1}{s + \frac{1}{RC}} \cdot \frac{1}{s} \quad \left[H(s) = \frac{1}{RC} \frac{1}{s + \frac{1}{RC}} \Rightarrow L^{-1}\{H(s)\} = \frac{1}{\tau} e^{-\frac{t}{\tau}} (\tau = RC) \right]$$



$$u_c(t) = \int_0^t \frac{1}{\tau} e^{-\frac{x}{\tau}} dx = \left(-e^{-\frac{x}{\tau}} \right) \Big|_{x=0}^{x=t} = 1 - e^{-\frac{t}{\tau}} \quad (t \geq 0)$$

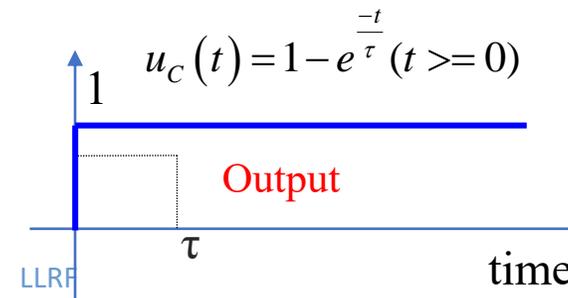
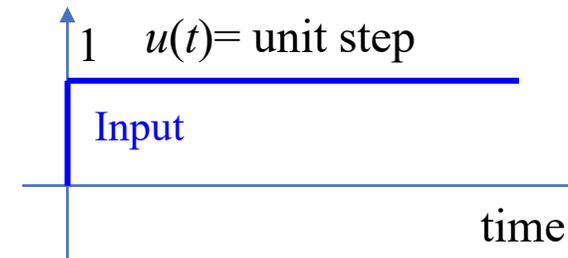
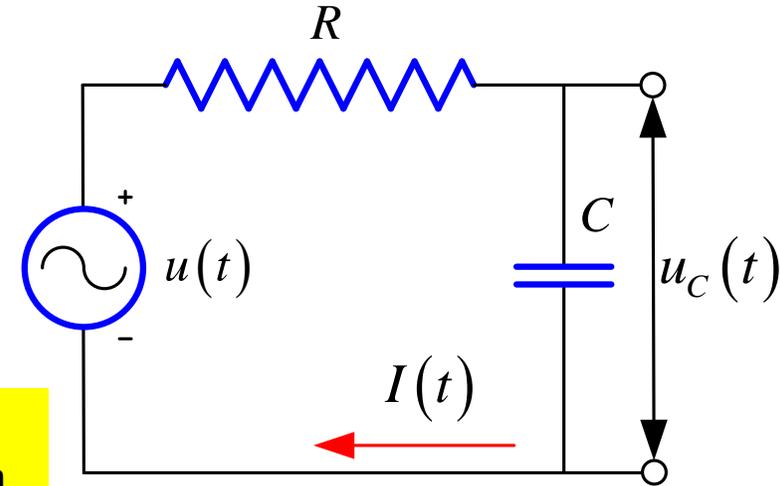
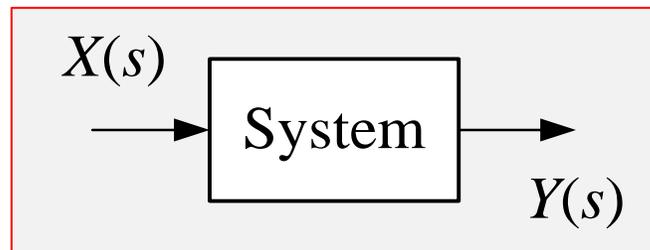
Example: RC circuits

- Go back to the differential equation:

$$\frac{du_c(t)}{dt} + \frac{u_c(t)}{RC} = \frac{u(t)}{RC}$$

Solution of the differential equation

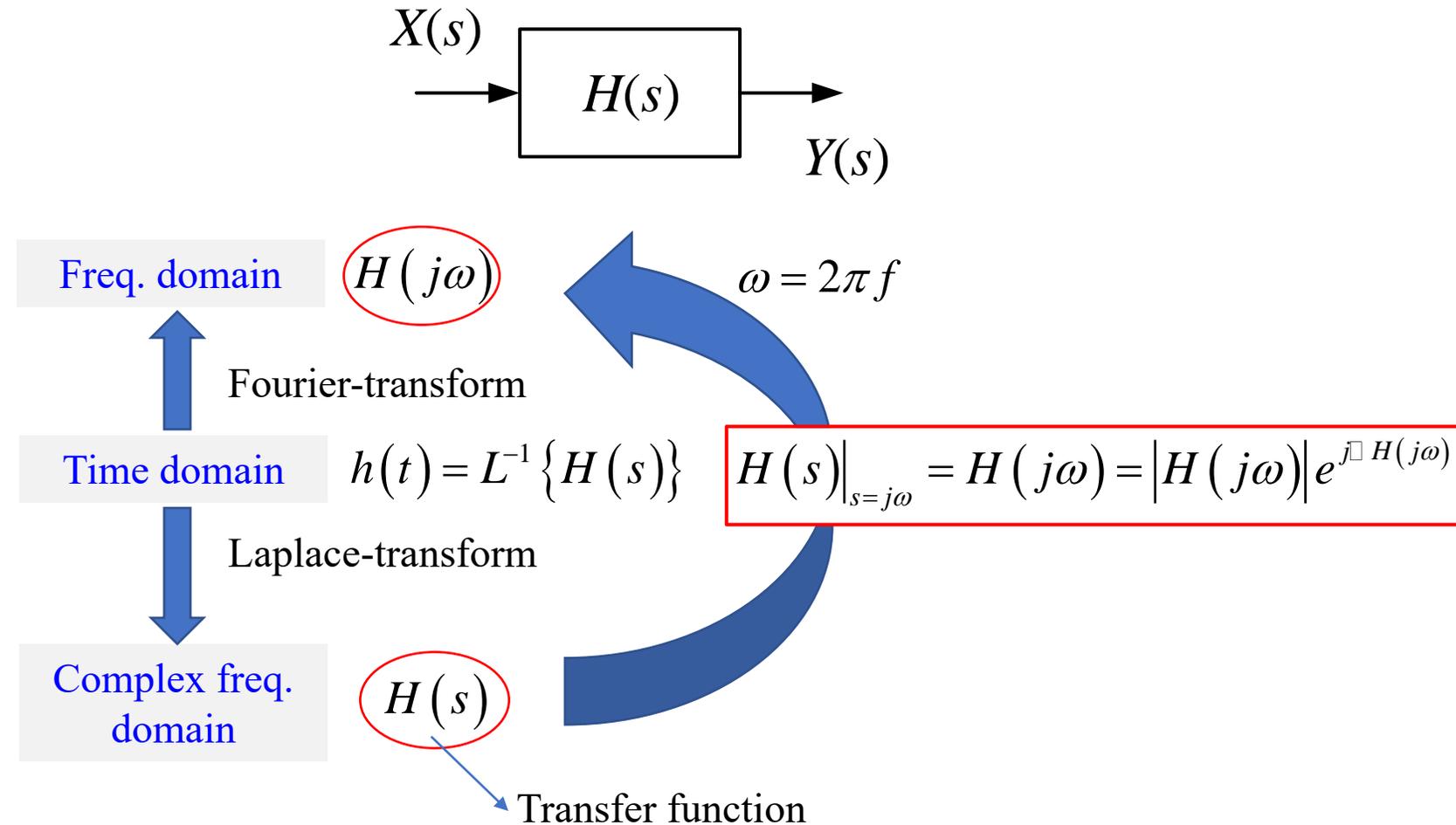
$$u_c(t) = 1 - e^{-\frac{t}{\tau}} \quad (t \geq 0)$$



Usually, we also call the parameter τ time constant.

Frequency response

- It is a measure of magnitude and phase of the output as a function of frequency.
- From TF to transfer function: $H(s) \rightarrow H(j\omega)$.



Frequency response

- If we know $H(j\omega)$, we also know $H(j2\pi \cdot f)$
- And then $A(f)$ & $P(f)$

$$H(s)\Big|_{s=j\omega} = H(j\omega) = |H(j\omega)| e^{j\angle H(j\omega)} = |H(j2\pi f)| e^{j\angle H(j2\pi f)}$$

Amplitude vs. frequency

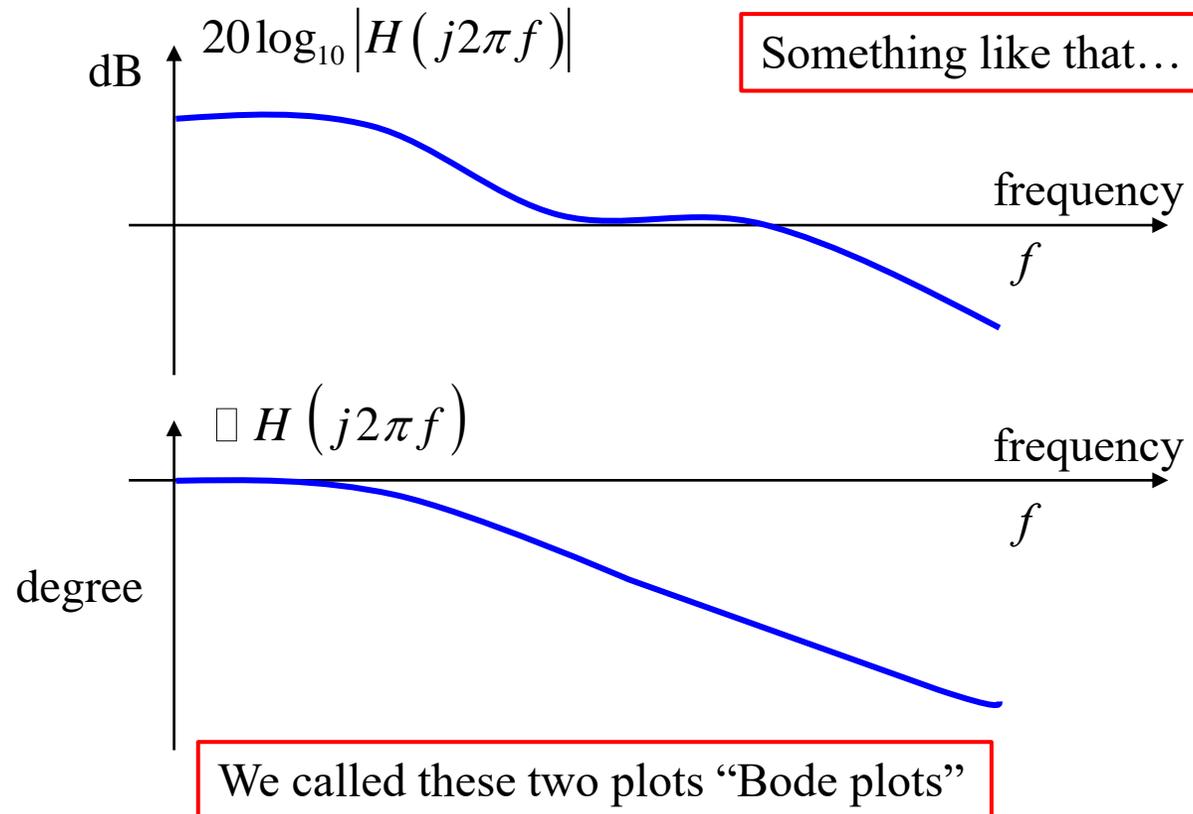
Phase vs. frequency

- We can also plot $A(f)$ & $P(f)$, if we know their expressions

Frequency response (Bode plot)

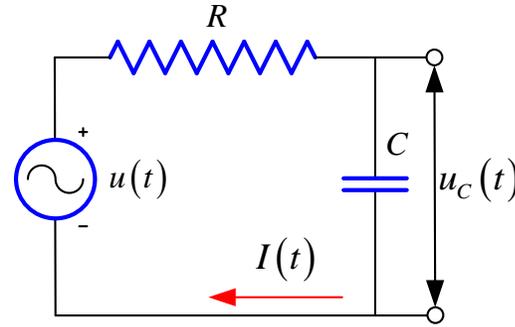
- Bode diagram: plots of the amplitude-frequency and phase-frequency response of the system $H(s)$.

$$H(s)\Big|_{s=j\omega} = H(j\omega) = |H(j\omega)| e^{j\angle H(j\omega)} = |H(j2\pi f)| e^{j\angle H(j2\pi f)}$$



Bode diagram

- Bode diagram: plots of the amplitude-frequency and phase-frequency response of the system $H(s)$.

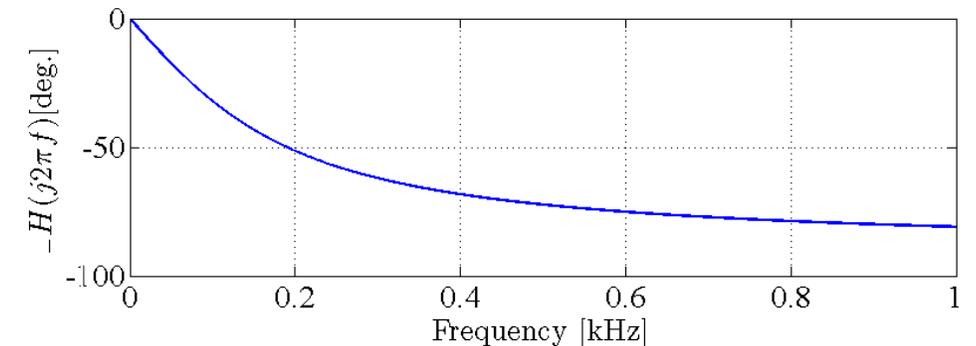
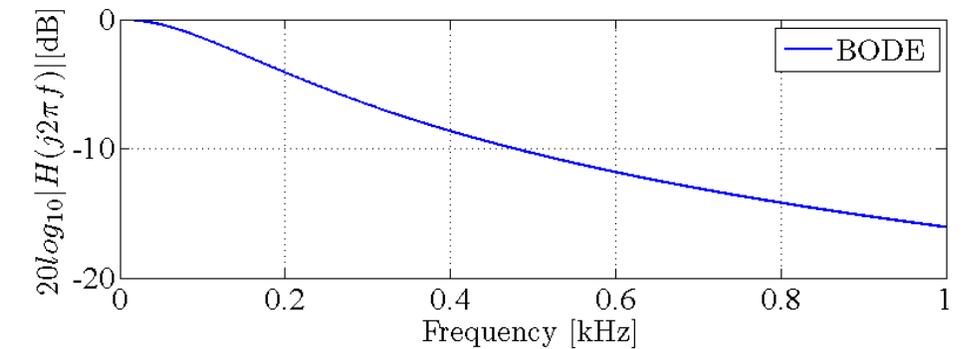


Amplitude vs. frequency

Phase vs. frequency

$$|H(j2\pi f)| = \frac{\frac{1}{\tau}}{\sqrt{(2\pi f)^2 + \left(\frac{1}{\tau}\right)^2}}, \text{ and } \angle H(j2\pi f) = -\arctan(2\pi f\tau)$$

Here $\tau = 1 \text{ ms}$



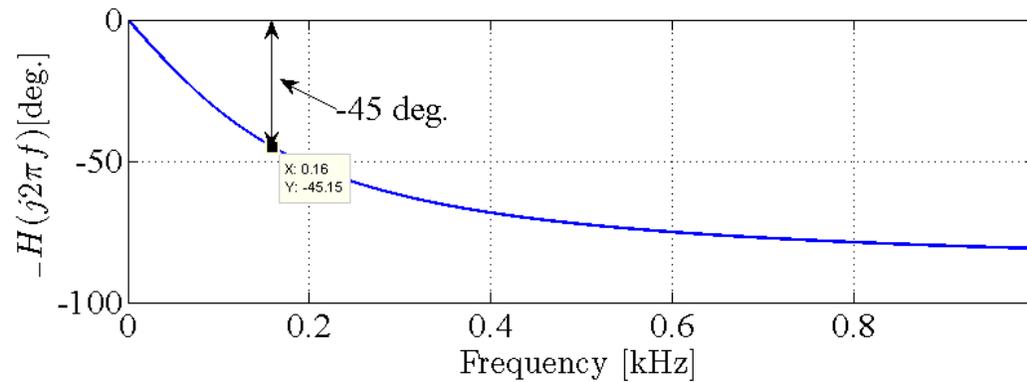
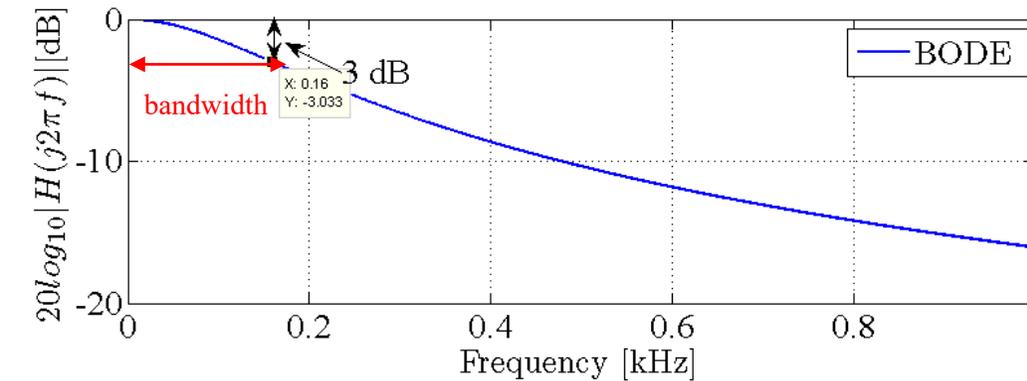
Bode diagram

How about some special case,
 $\omega = 1/\tau = 1000$ [rad/s]. $f=160$ Hz

$$\left| H\left(j\frac{1}{\tau}\right) \right| = \frac{\frac{1}{\tau}}{\sqrt{\left(\frac{1}{\tau}\right)^2 + \left(\frac{1}{\tau}\right)^2}} = \frac{1}{\sqrt{2}}$$

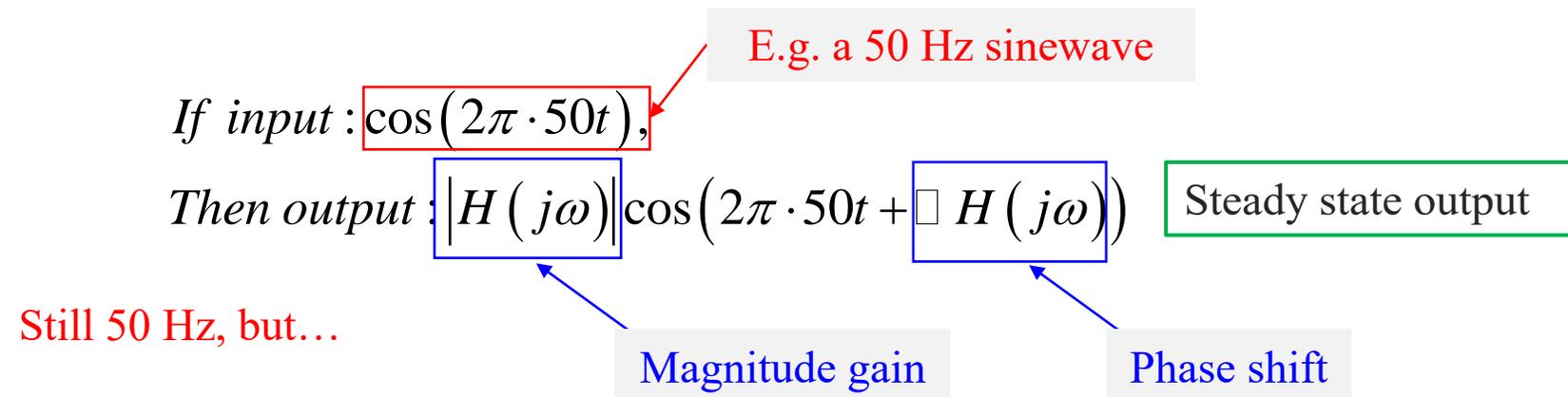
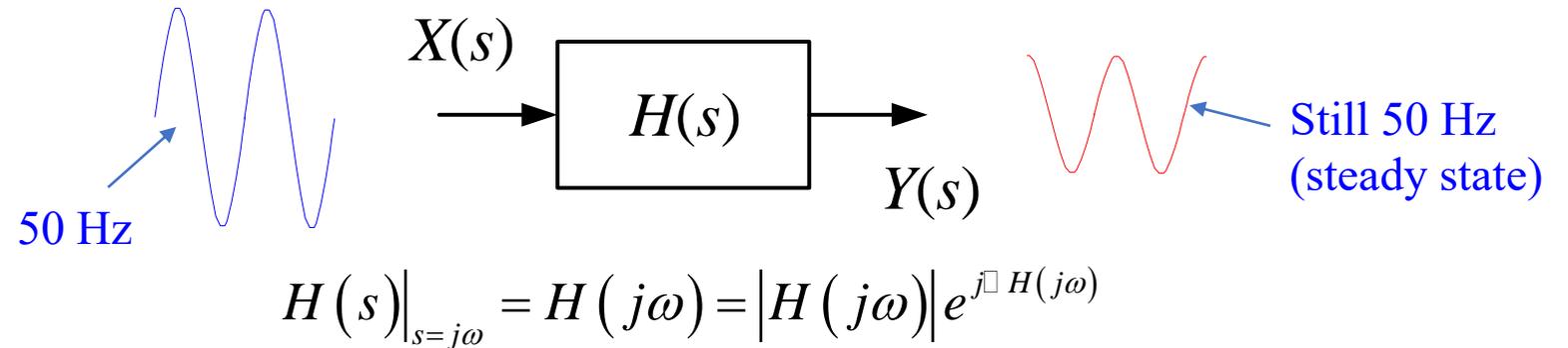
$$\square H\left(j\frac{1}{\tau}\right) = -\arctan\left(\frac{1}{\tau}\right) = -45^\circ$$

The **half power point** is that frequency at which the output **power (not voltage)** has dropped to half of its mid-band value.



Frequency response

- If we know the frequency response of a system or its bode diagram, then let's consider a sinusoidal signal



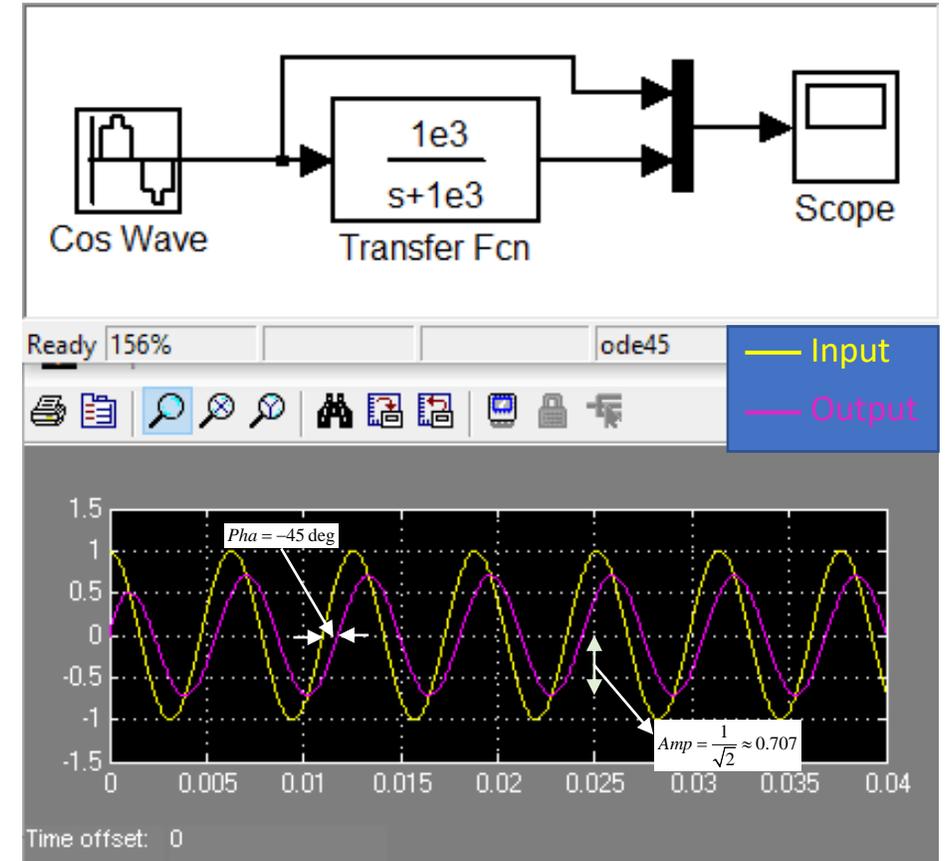
Frequency response

- We can simulate this with using Matlab/Simulink
- The input is a 160 Hz sinusoidal signal

Special case,
 $\omega = 1/\tau = 1000$ [rad/s].
 $f = \omega/2\pi = 160$ Hz

$$\left| H\left(j\frac{1}{\tau}\right) \right| = \frac{\frac{1}{\tau}}{\sqrt{\left(\frac{1}{\tau}\right)^2 + \left(\frac{1}{\tau}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\angle H\left(j\frac{1}{\tau}\right) = -\arctan(1) = -45^\circ$$

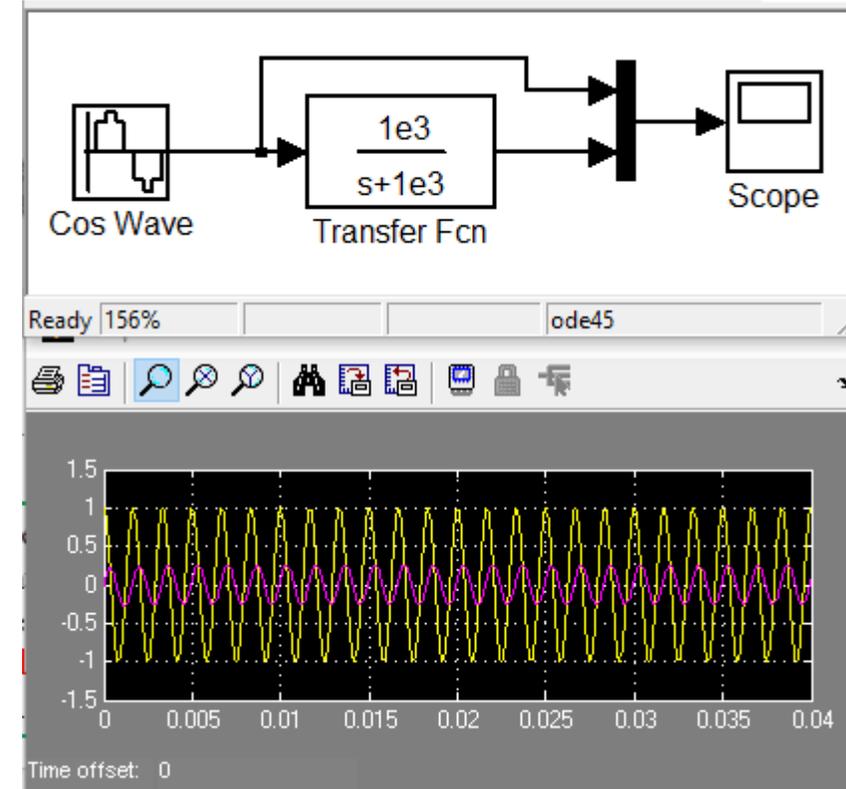
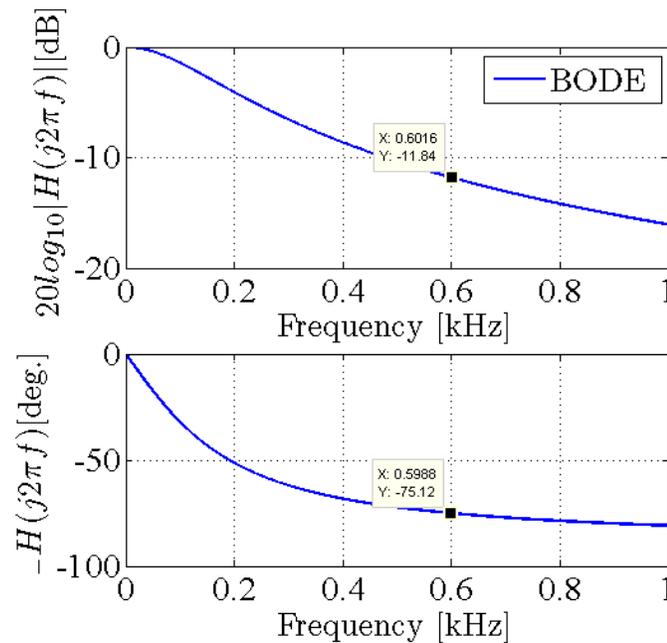


If input : $\cos(2\pi \cdot 160t)$,

$$\text{Then output : } |H(j\omega)| \cos(2\pi \cdot 160t + \angle H(j\omega)) = \frac{1}{\sqrt{2}} \cos\left(2\pi \cdot 160t - \frac{\pi}{4}\right)$$

Frequency response

- How about $f = 600 \text{ Hz}$?

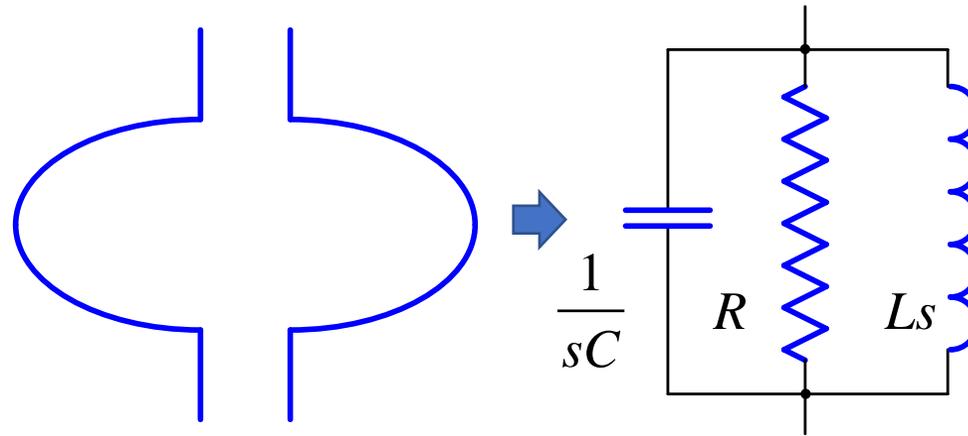


If input: $\cos(2\pi \cdot 600t)$,

Then output: $|H(j\omega)| \cos(2\pi \cdot 600t + \angle H(j\omega)) \approx \frac{1}{4} \cos\left(2\pi \cdot 600t - \frac{5\pi}{12}\right)$

Example of cavity

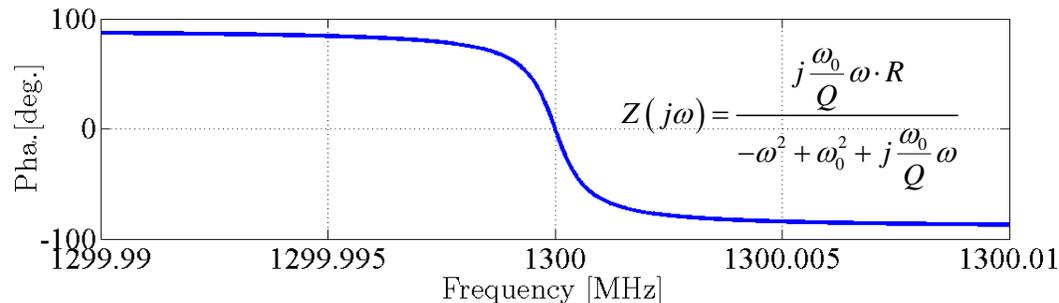
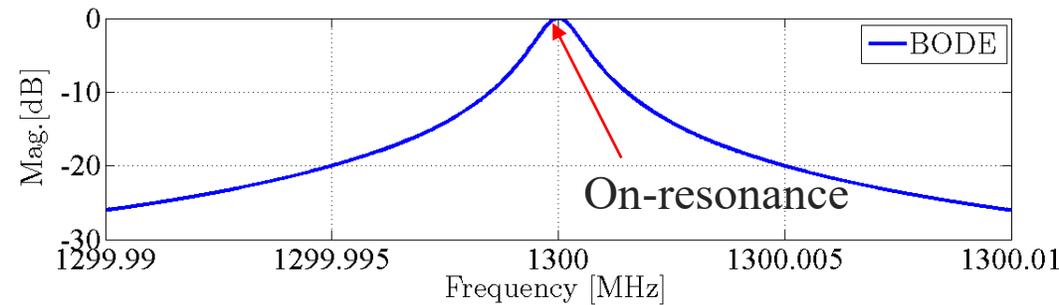
- Bode diagram (Plots of the amplitude-frequency and phase-frequency response of the system)



$$Z(s) = \frac{\frac{s}{C}}{s^2 + \frac{1}{RC}s + \omega_0^2} = \frac{\frac{R\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

E.g. $\omega_0 = 2\pi f_0 = 2\pi(1.3 \times 10^9) [\text{rad} / \text{s}]$

$Q = 1.3 \times 10^6$



Bode Plot

- Magnitude vs. frequency
- Phase vs. frequency

Bode plot (SC vs. NC)

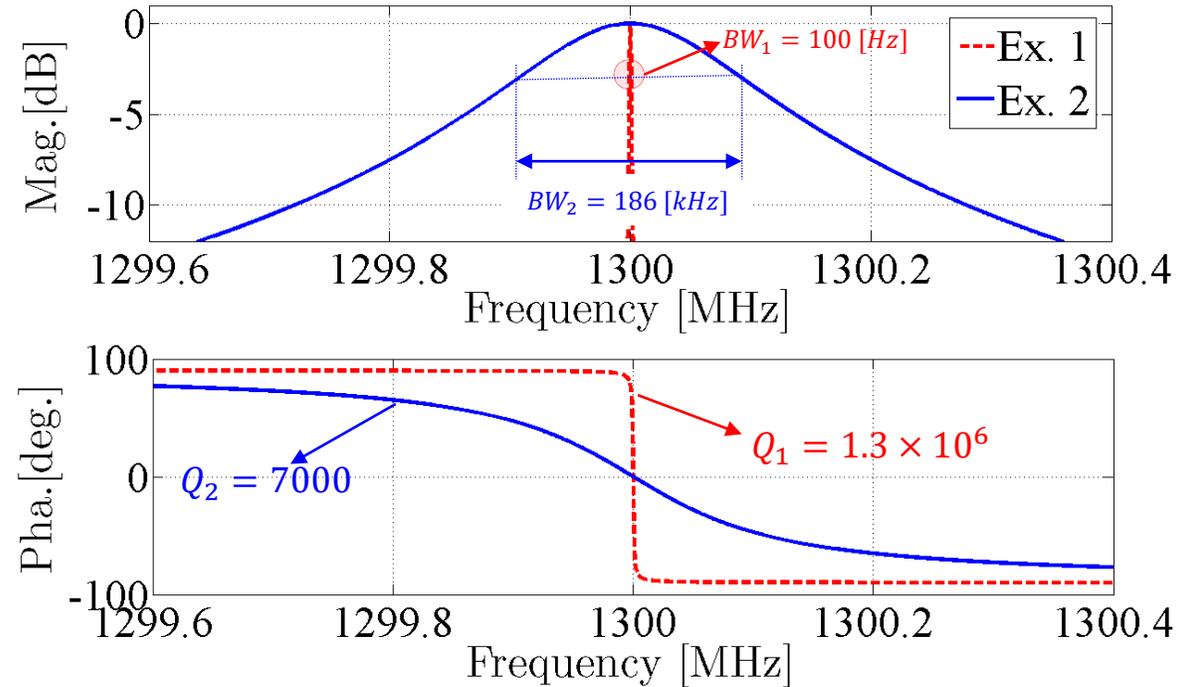
Ex. 1 Bode Plot (SC)

- $\omega_0 = 1.3 \cdot 2\pi$ [rad/s]
- $Q_1 = 1.3 \times 10^6$
- $BW_1 = 100$ [Hz]

We can clearly see that the bandwidth is related to the Q value.

Ex. 2 Bode Plot (NC)

- $\omega_0 = 1.3 \cdot 2\pi$ [rad/s]
- $Q_2 = 7000$
- $BW_2 = 186$ [kHz]

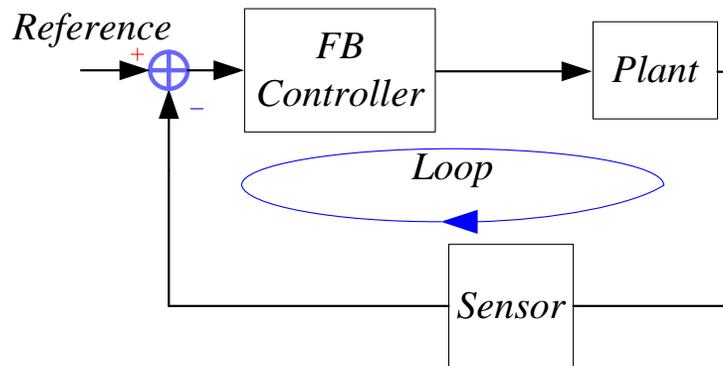


Resonance frequency of a cavity (1.3 GHz)

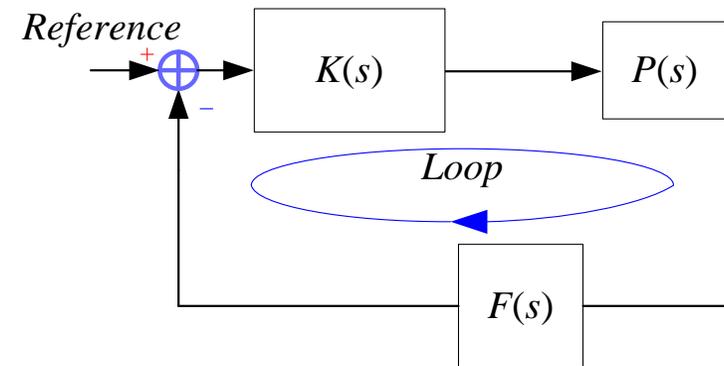
$$BW_{3dB} = \frac{f_0}{Q}, f_0 = \frac{\omega_0}{2\pi}, \text{Half } BW(\omega_{0.5}) = \frac{f_0}{2Q}$$

Transfer function of a FB system

- Let us go back to the basic control system, now let's review it in the viewpoint of the transfer functions.



Basic control system



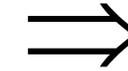
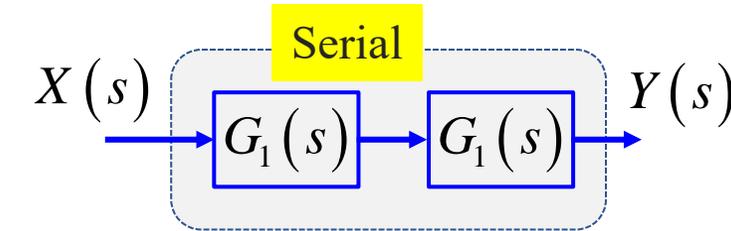
TF representation

LLRF system

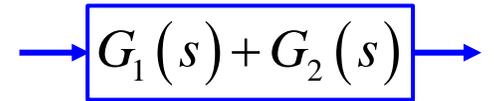
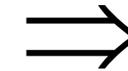
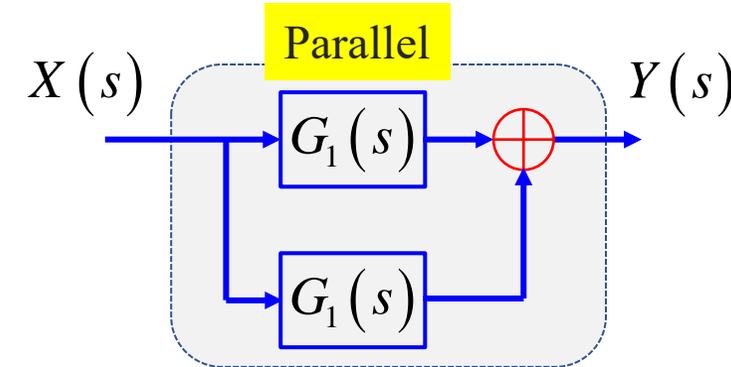
- $K(s)$: Controller. generally, a proportional & integral (PI) controller
- $P(s)$: Plant you want to control
- $F(s)$: Detector (sensor), to measured the response of the plant

Block diagram transformations

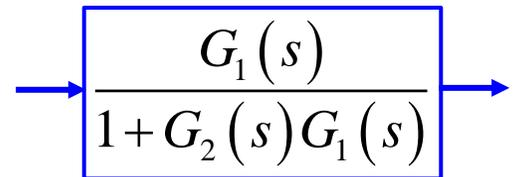
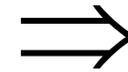
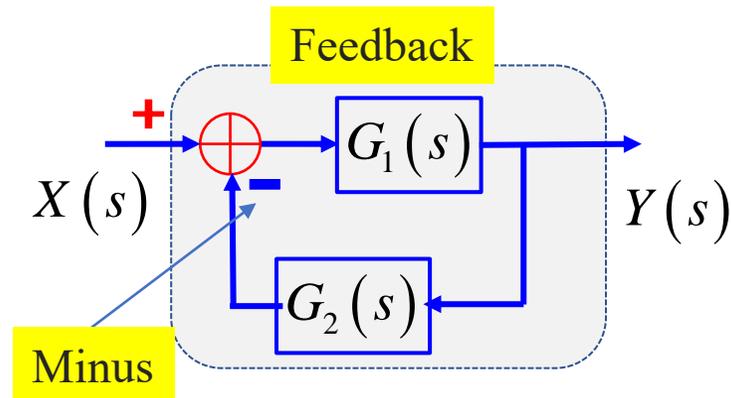
- How to calculate the transfer function of the whole system, if we know the transfer function of each subsystem?



$$\frac{Y(s)}{X(s)} = G_2(s)G_1(s)$$



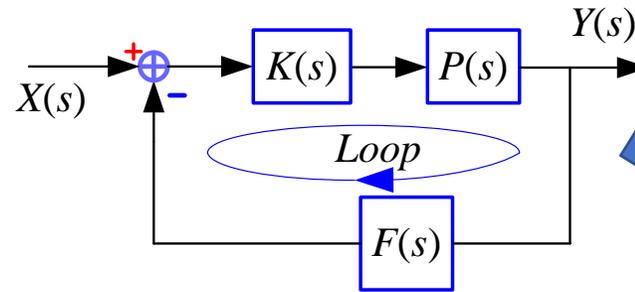
$$\frac{Y(s)}{X(s)} = G_1(s) + G_2(s)$$



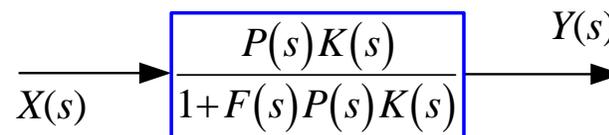
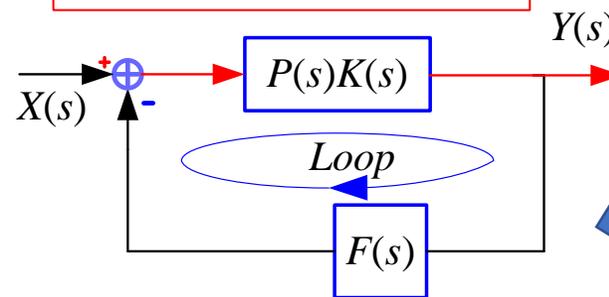
$$\frac{Y(s)}{X(s)} = \frac{G_1(s)}{1 + G_2(s)G_1(s)}$$

Example

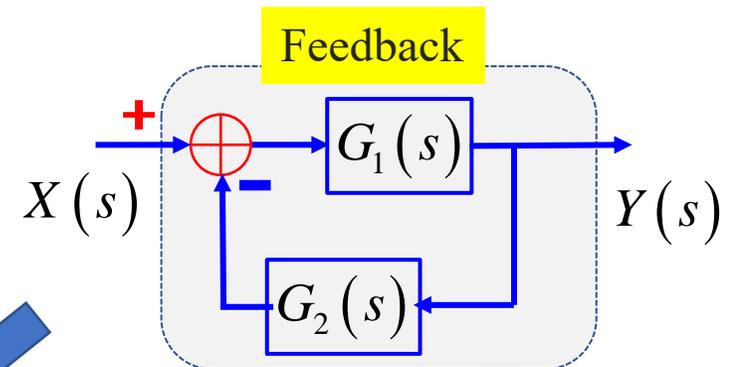
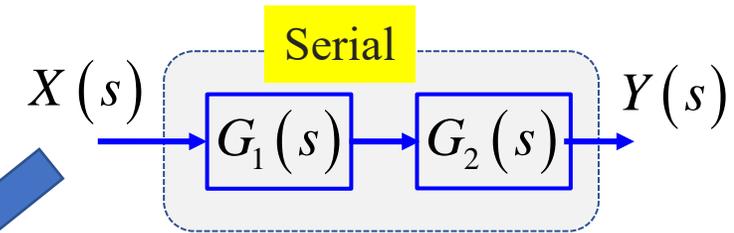
- How to calibrate the transfer function from $X(s)$ to $Y(s)$?



Forward gain = $P(s)K(s)$



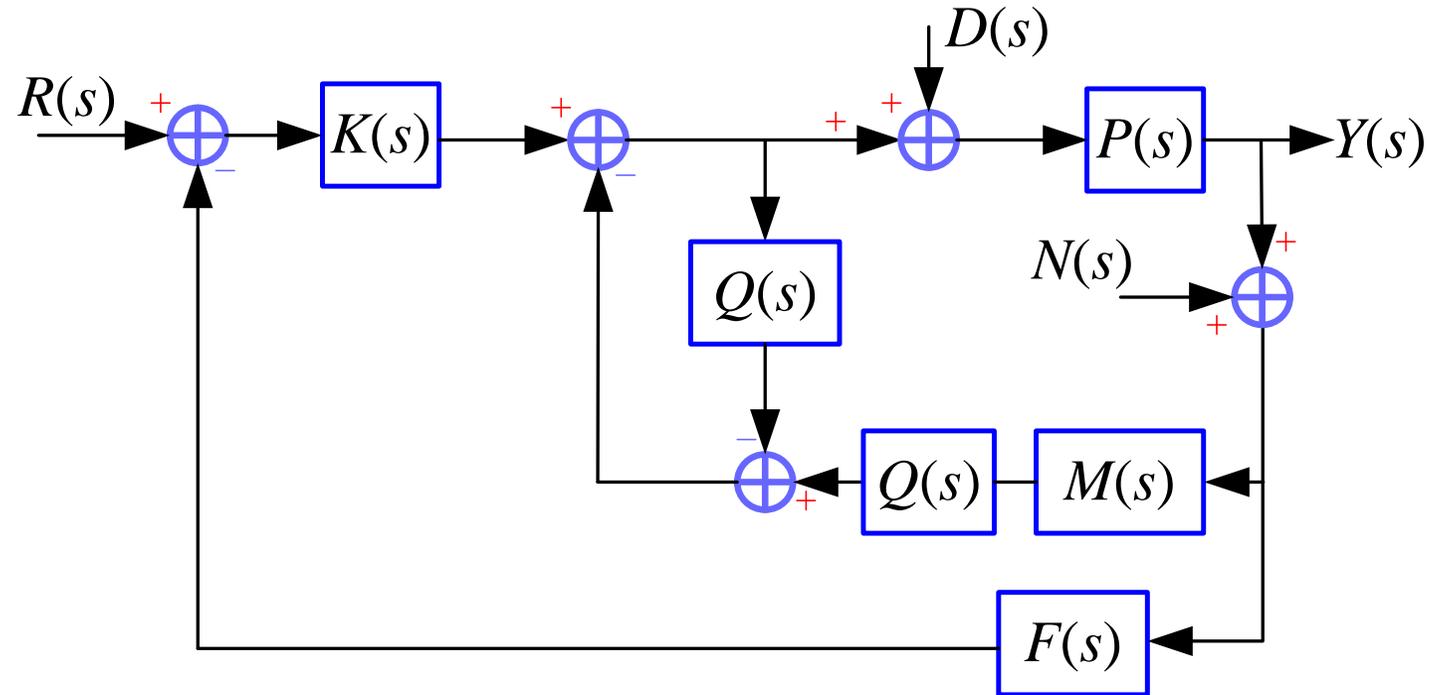
$$\frac{Y(s)}{X(s)} = G_2(s)G_1(s)$$



$$\frac{Y(s)}{X(s)} = \frac{G_1(s)}{1 + G_2(s)G_1(s)} = \frac{\text{forward gain}}{1 + \text{open loop gain}}$$

Mason's rule

- To go a little bit further. In some cases, to calculate the transfer function is not so easy, for example (too many loops):

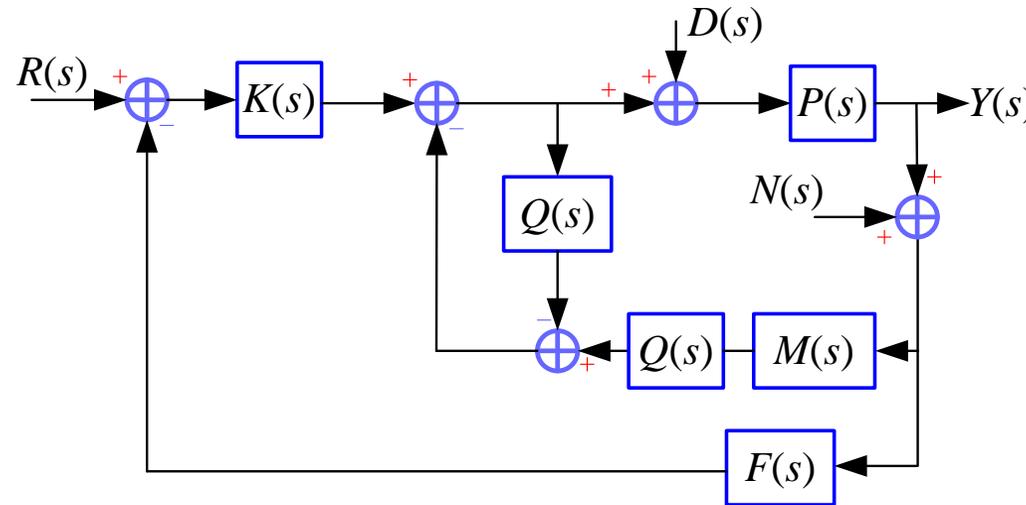


$$H_{d \rightarrow y} = \frac{Y(s)}{D(s)} = ?$$

$$H_{n \rightarrow y} = \frac{N(s)}{D(s)} = ?$$

$$H_{r \rightarrow y} = \frac{R(s)}{D(s)} = ?$$

Mason's rule



Mason's Gain Rule:

$$M = \frac{\sum_j M_j \Delta_j}{\Delta}$$

M = transfer function or gain of the system

M_j = gain of one forward path

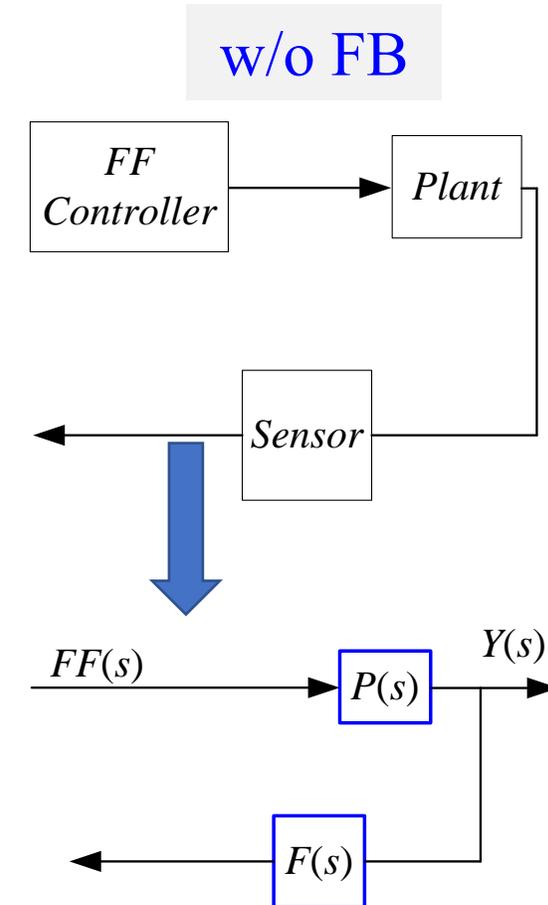
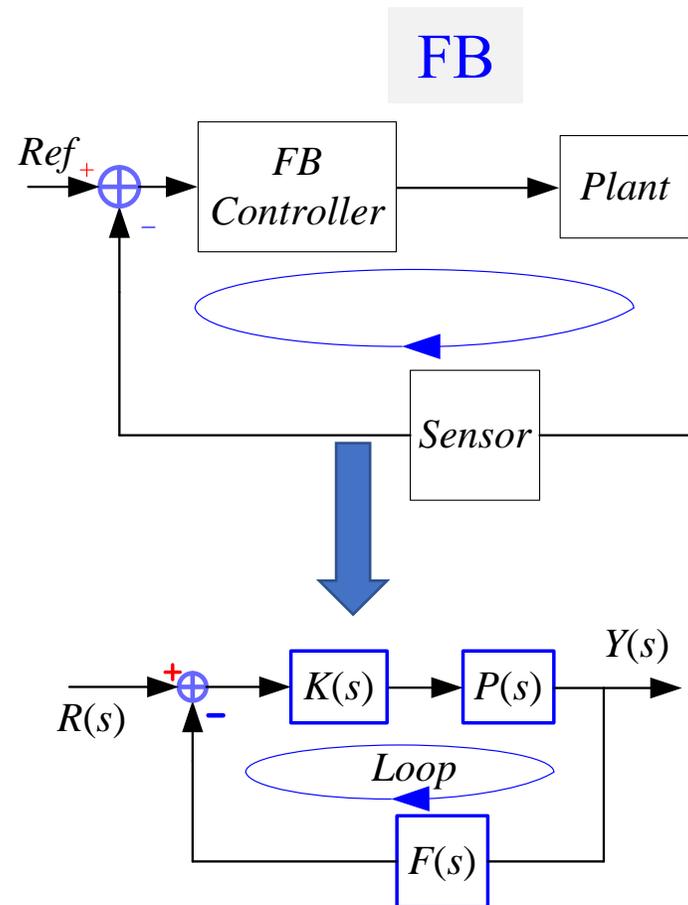
j = an integer representing the forward paths in the system

$\Delta_j = 1 -$ the loops remaining after removing path j . If none remain, then $\Delta_j = 1$.

$\Delta = 1 - \Sigma$ loop gains + Σ nontouching loop gains taken two at a time - Σ nontouching loop gains taken three at a time + Σ nontouching loop gains taken four at a time - \dots

With FB vs. w/o FB

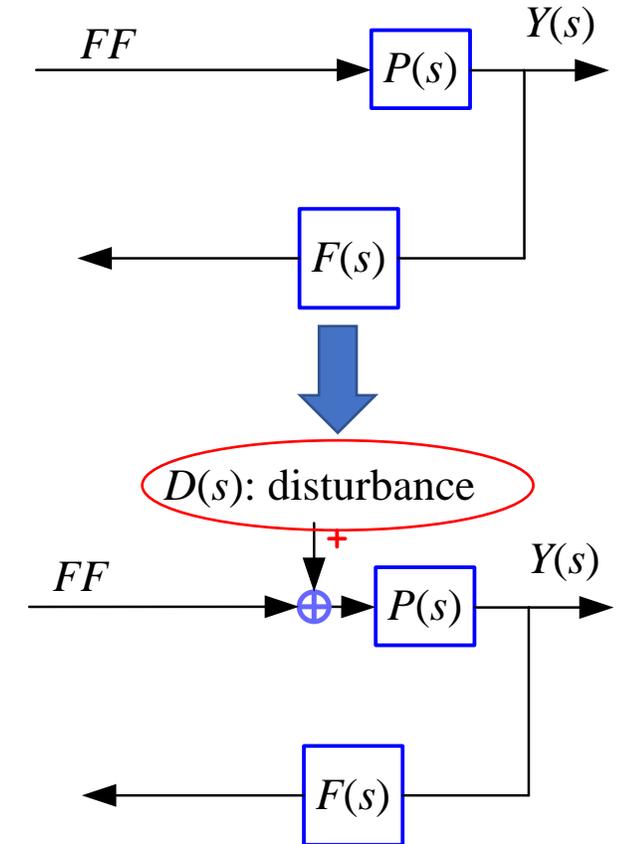
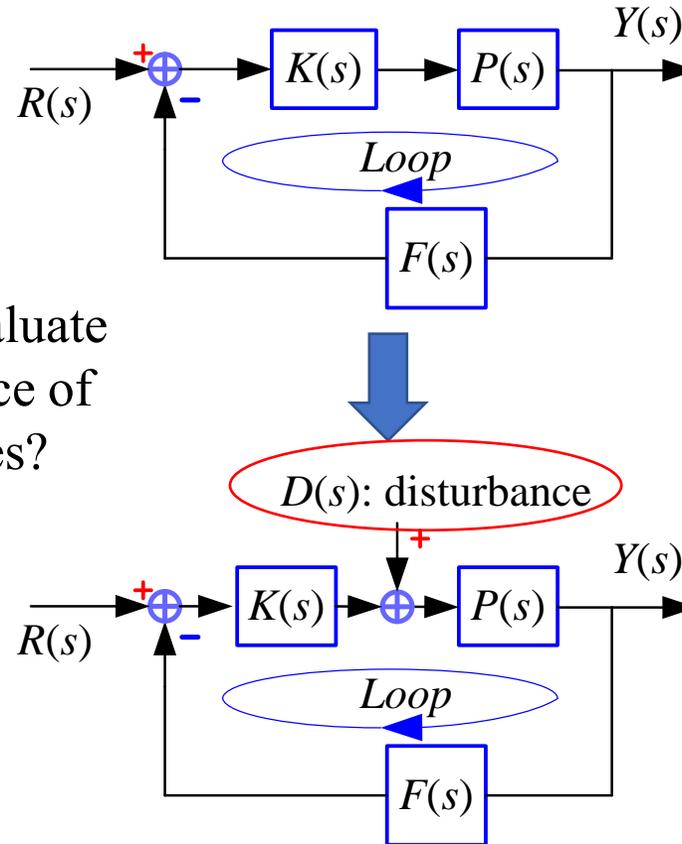
- In the following let's find an answer to the question: "Why do we need feedback?"
- We will do so by evaluating the transfer functions



Disturbance Rejection

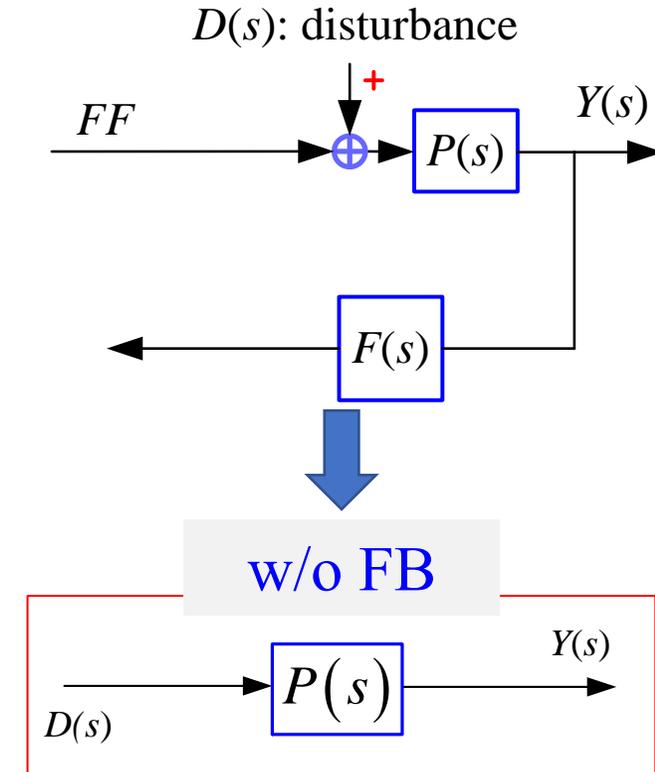
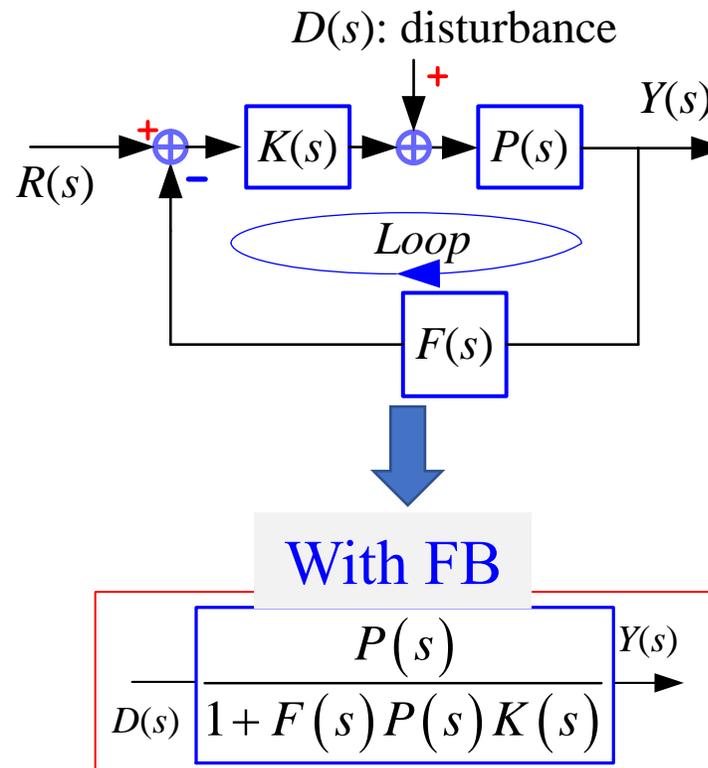
- In the real case, disturbances exists in the system (not only LLRF system, but almost al of the control system), so the actual system is something like:

How to evaluate the influence of disturbances?



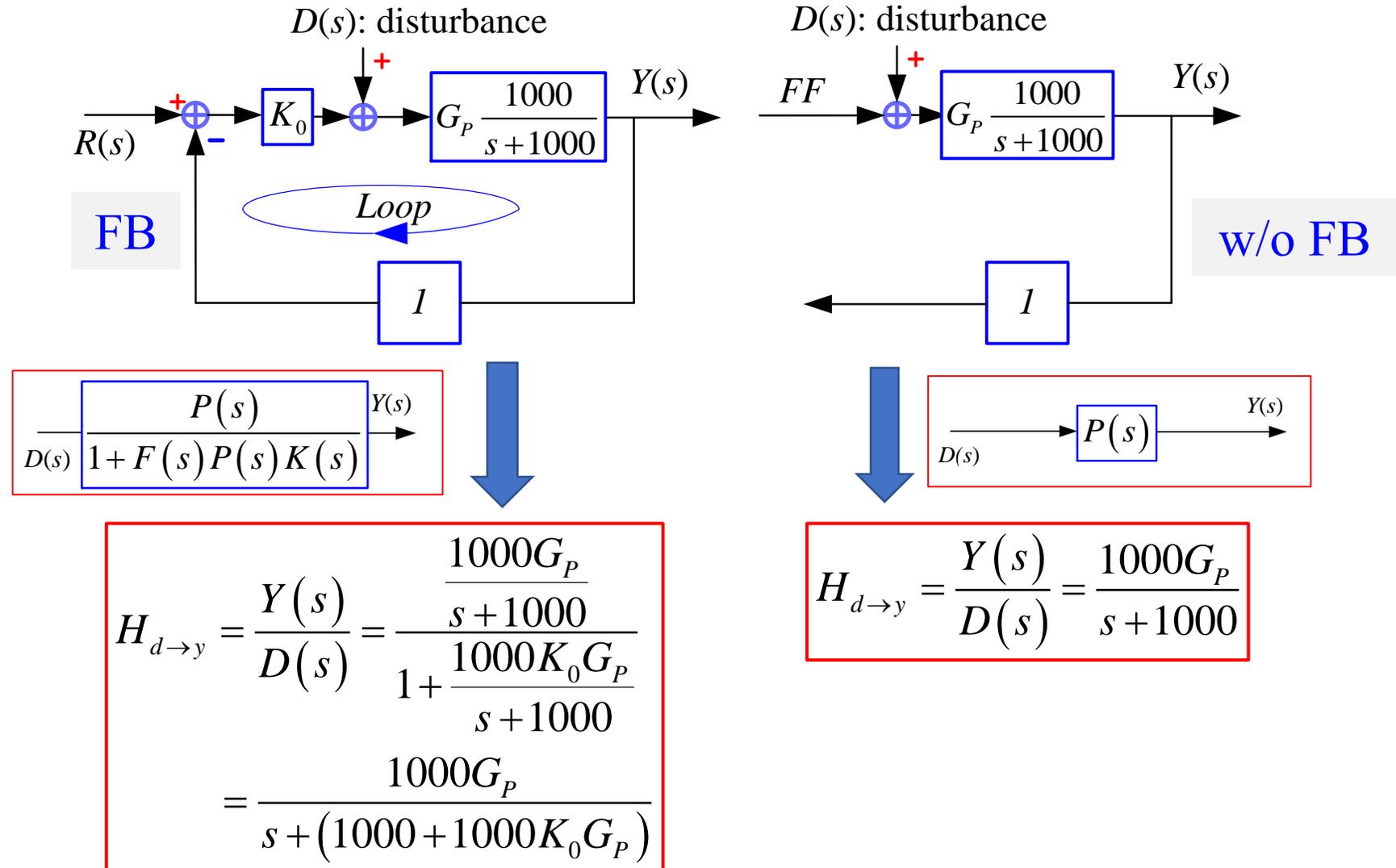
Disturbance Rejection

- Obviously, the existence of the disturbance (or perturbation) will influence the system, but is it same for FB and FF?



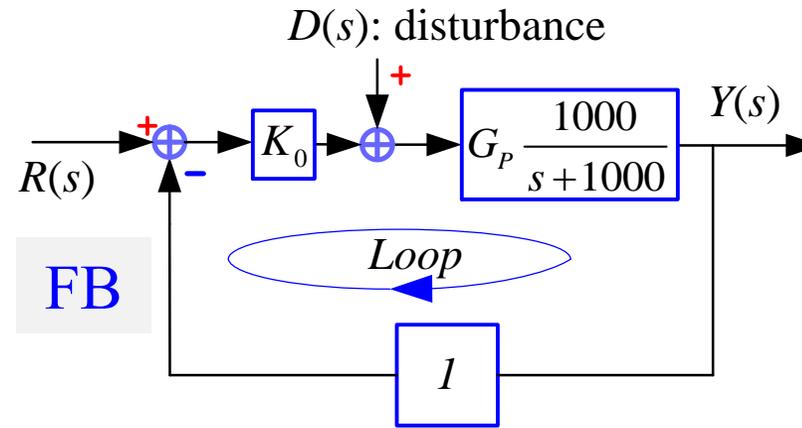
Disturbance Rejection

- Let's give each component ($H(s)$) some meaning



Disturbance Rejection

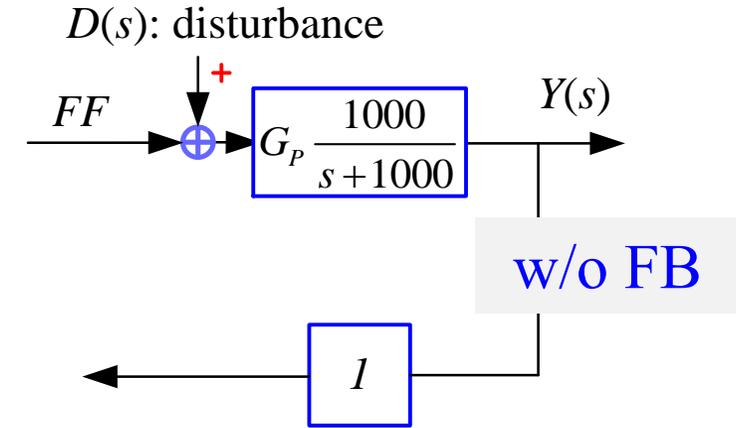
- Furthermore, if $G_P = 1$, then



$$H_{d \rightarrow y}(s) = \frac{1000}{s + (1000 + 1000K_0)}$$

$$H_{d \rightarrow y}(j\omega) = \frac{1000}{j\omega + (1000 + 1000K_0)}$$

Considering the $H(j\omega)$



$$H_{d \rightarrow y}(s) = \frac{Y(s)}{D(s)} = \frac{1000}{s + 1000}$$

$$H_{d \rightarrow y}(j\omega) = \frac{1000}{j\omega + 1000}$$

Disturbance Rejection

- The best way is to compare their frequency response or bode plot?
- Bode diagram: plots of the amplitude-frequency and phase-frequency response of the system $H(s)$.

FB

$$H_{d \rightarrow y}(j\omega) = \frac{1000}{j\omega + (1000 + 1000K_0)}$$

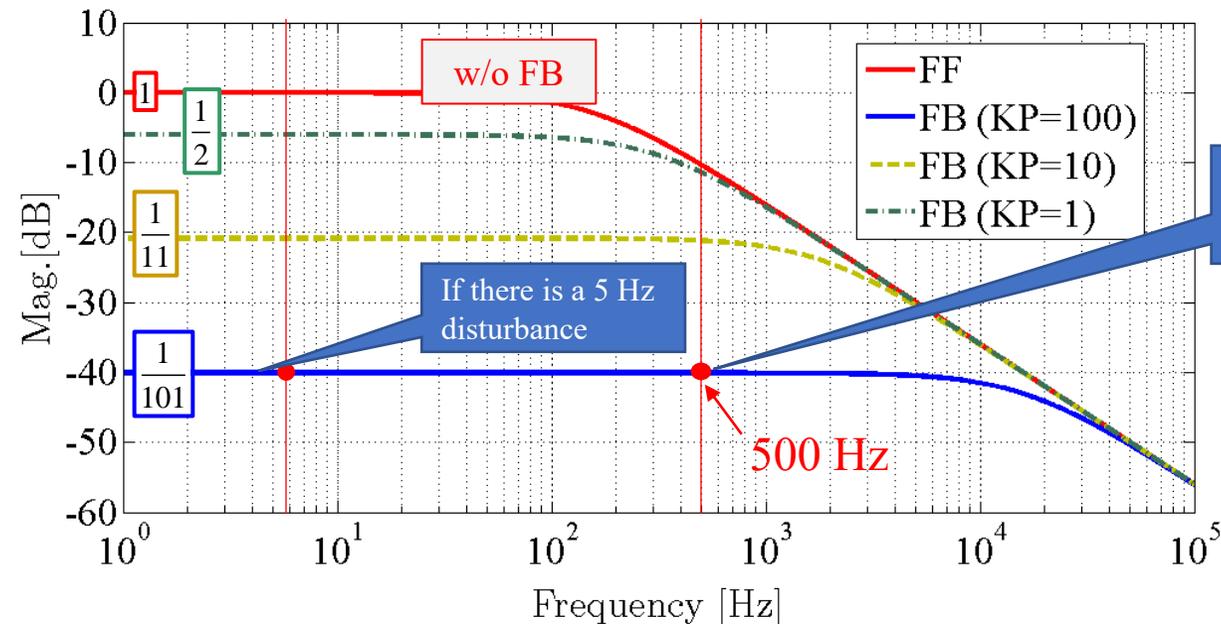
w/o FB

$$H_{d \rightarrow y}(j\omega) = \frac{1000}{j\omega + 1000}$$

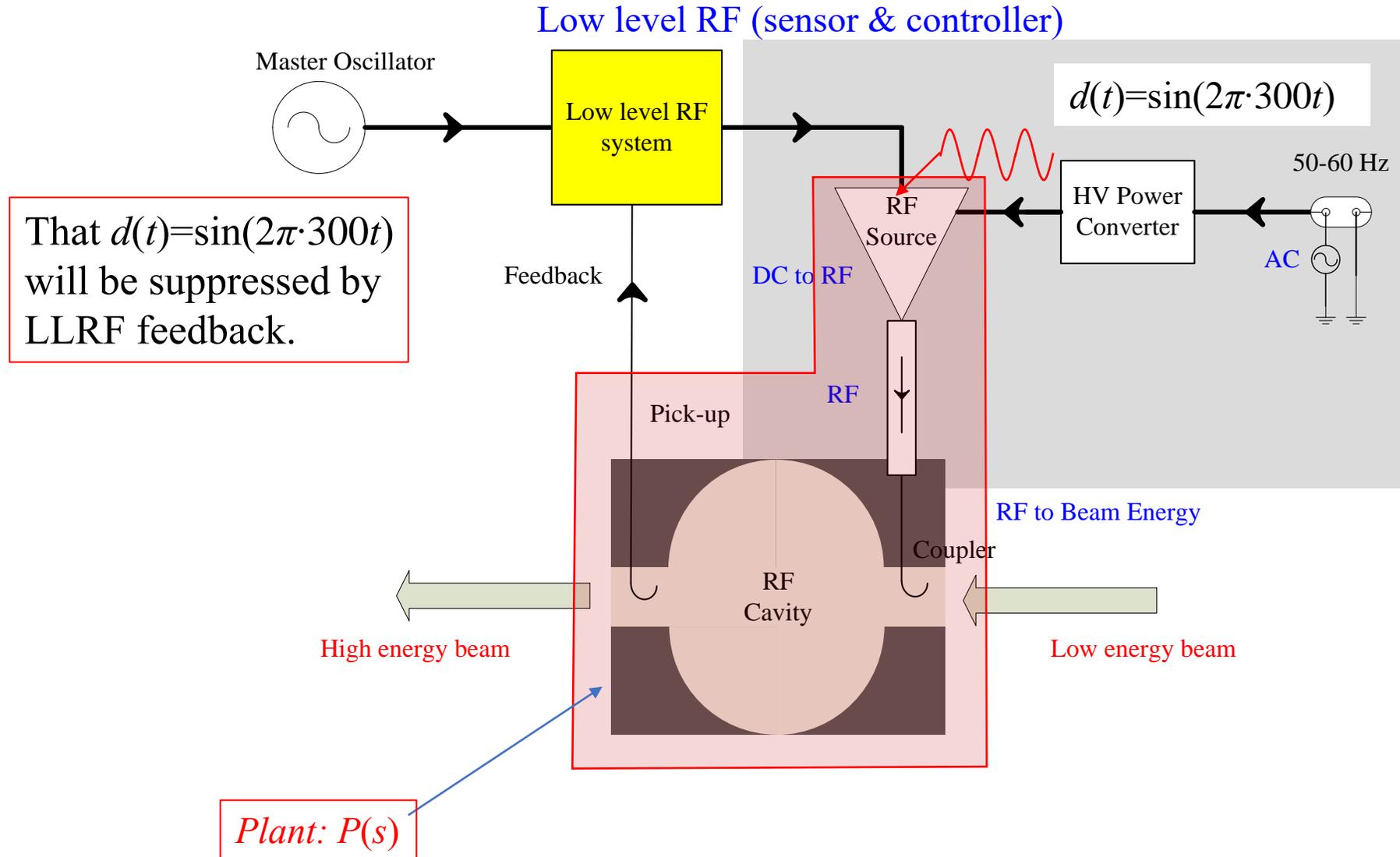
Let's find out the 0 Hz in the plot

$$H_{d \rightarrow y}(j \cdot 0) = \frac{1}{1 + K_0} \approx \frac{1}{K_0} (K_0 \gg 1)$$

$$H_{d \rightarrow y}(j \cdot 0) = 1$$

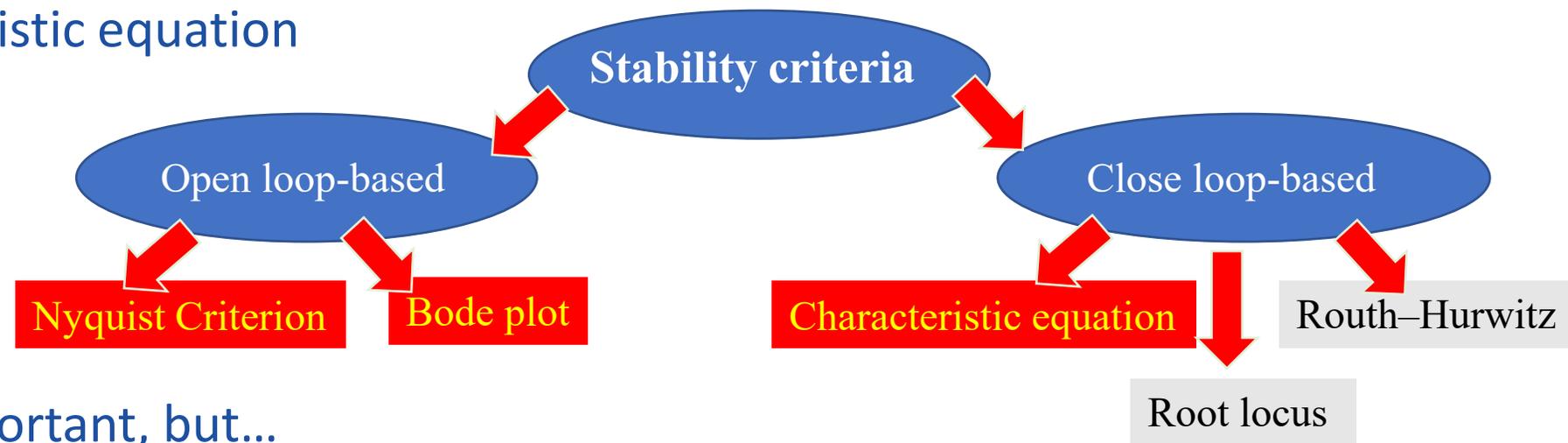


Power convertor ripples in RF system



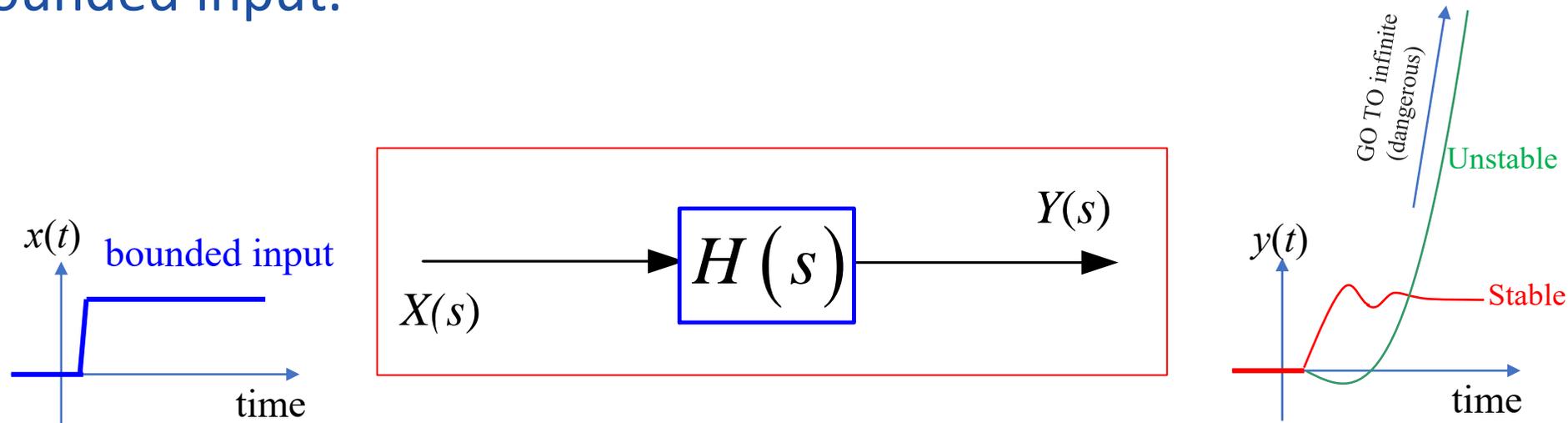
Stability Criteria

- Stability is most important for a feedback system. If the system is not stable, there is no meaning for any efforts.
- The Stability Criteria for a feedback system includes
 - Root locus
 - Solve the characteristic equation
 - Open loop bode plot & Nyquist Criterion
 - Routh–Hurwitz stability criterion
 - All of them are important, but...



Stability Criteria (poles position)

- Definition: A stable system is a dynamic system with a bounded response to a bounded input.

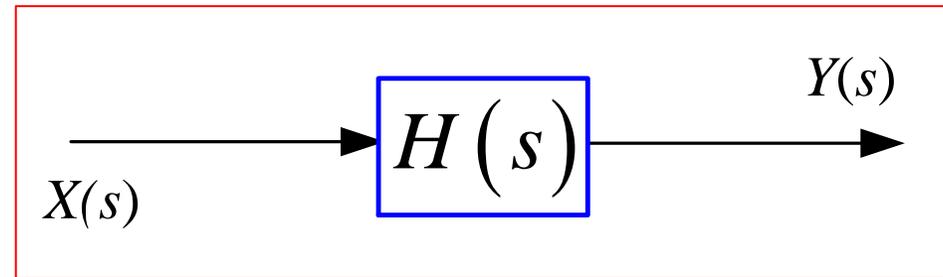


We can not try all of the bounded input signal

$$H(s) = \frac{\text{numerator}}{\text{denominator}} = \frac{b_m s^m + b_{m-1} s^{m-1} + b_{m-2} s^{m-2} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0}$$

Characteristic equation

Stability Criteria (poles position)



$$H(s) = \frac{\text{numerator}}{\text{denominator}} = \frac{b_m s^m + b_{m-1} s^{m-1} + b_{m-2} s^{m-2} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0}$$

$$= \frac{K(s - z_1)(s - z_2) \dots (s - z_{n-1})(s - z_n)}{(s - p_1)(s - p_2) \dots (s - p_{n-1})(s - p_n)}$$

$$H(s)|_{s=p} = \infty$$

Poles

$$H(s)|_{s=z} = 0$$

zeros

Complex number

A necessary and sufficient condition for a feedback system to be stable is that all the **poles** of the system transfer function have negative real parts.

Example

- The system $H(s)$ should be stable because all of the poles is in the have negative real part.

$$H(s) = \frac{s+1.5}{s^3 + 4s^2 + 6s + 4} = \frac{s - (-1.5)}{[s - (-1+i)] \cdot [s - (-1-i)] [s - (-2)]}$$

Three poles: $-1 \pm i, -2$

One zeros: -1.5

- The system $G(s)$ should be unstable because some poles have positive real part.

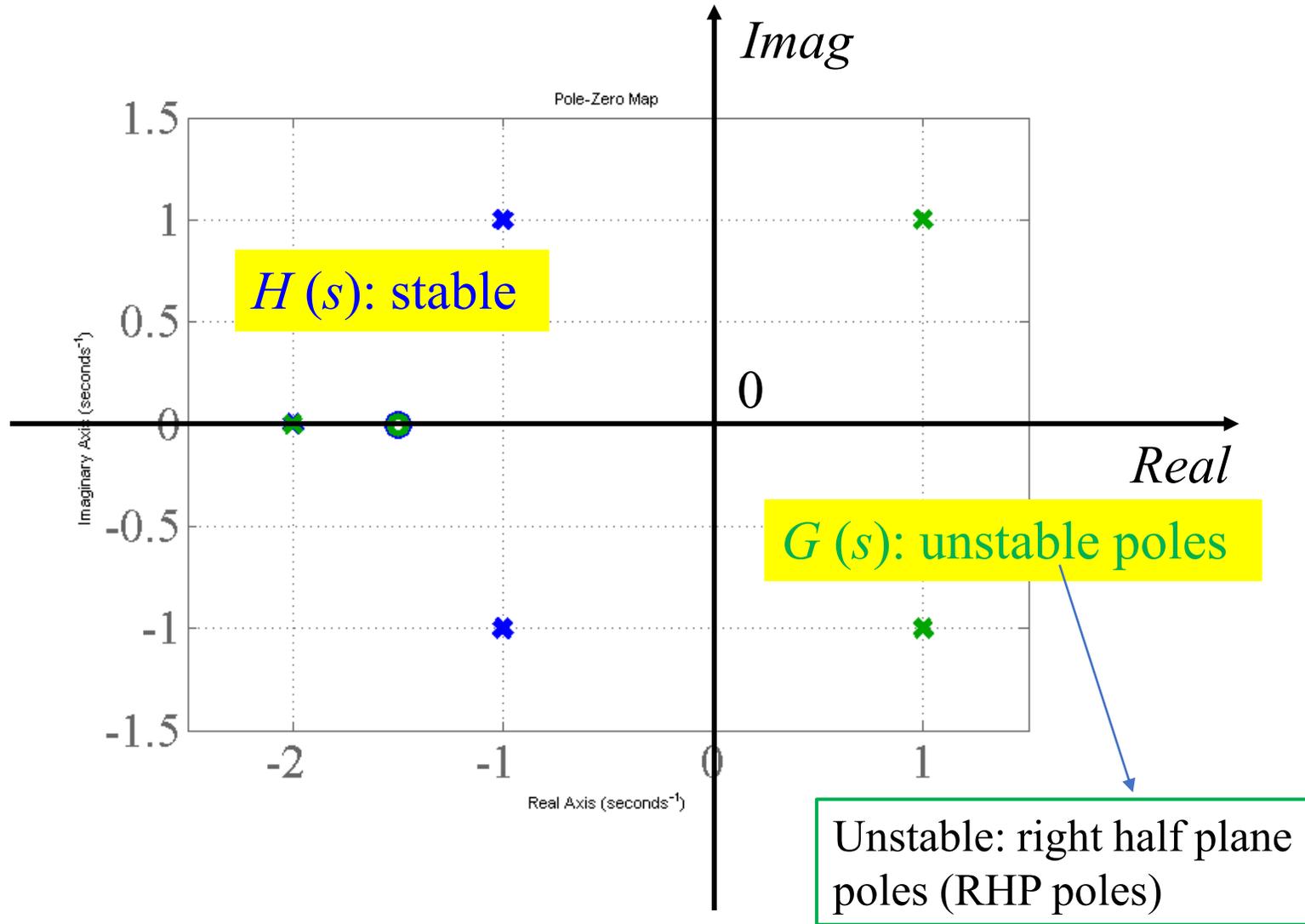
$$G(s) = \frac{s+1.5}{s^3 - 2s + 4} = \frac{s - (-1.5)}{[s - (1+i)] \cdot [s - (1-i)] [s - (-2)]}$$

positive real part

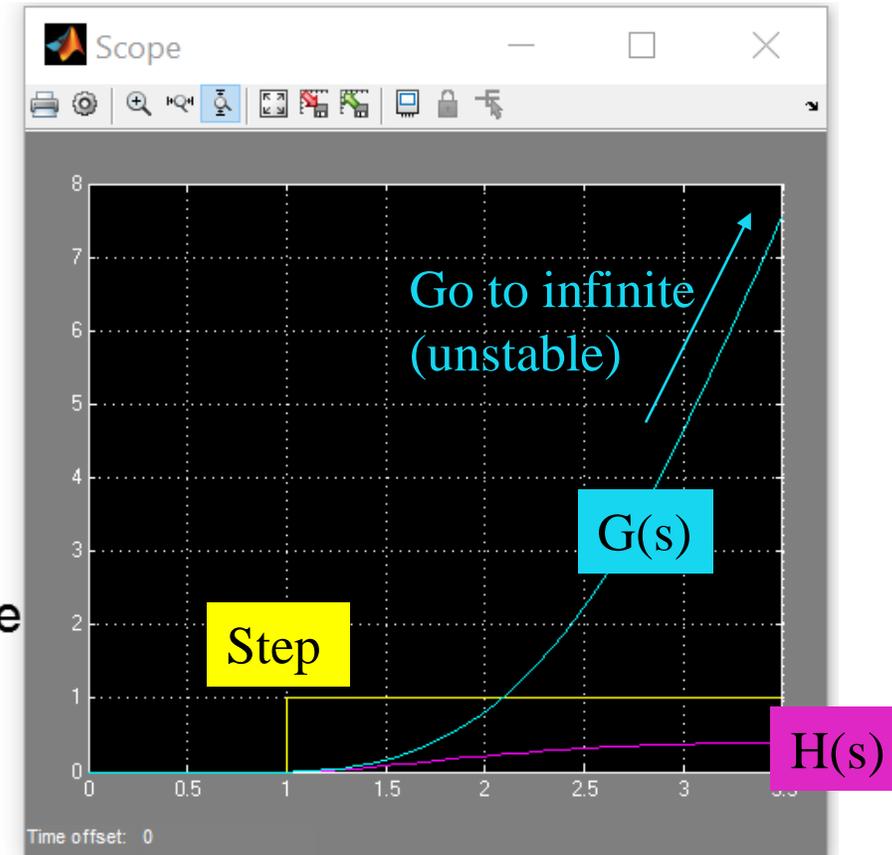
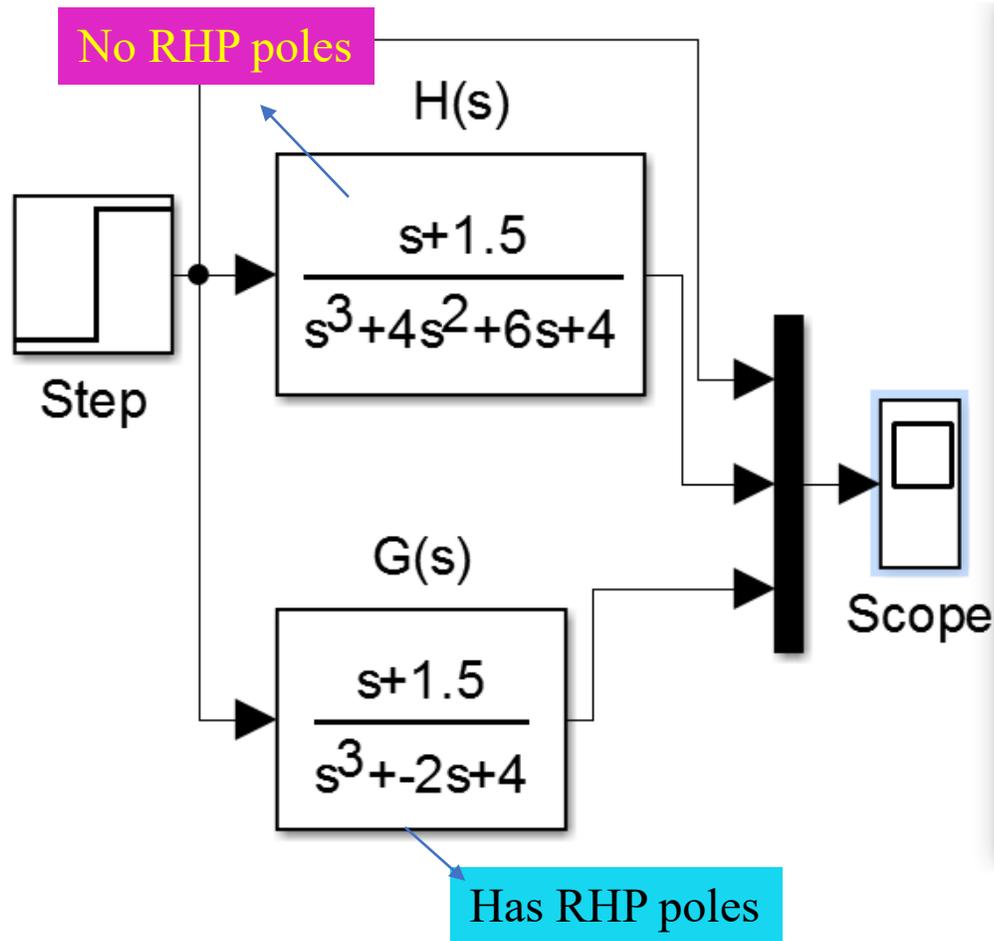
Three poles: $1 \pm i, -2$

One zeros: -1.5

Example (pole-zero map)

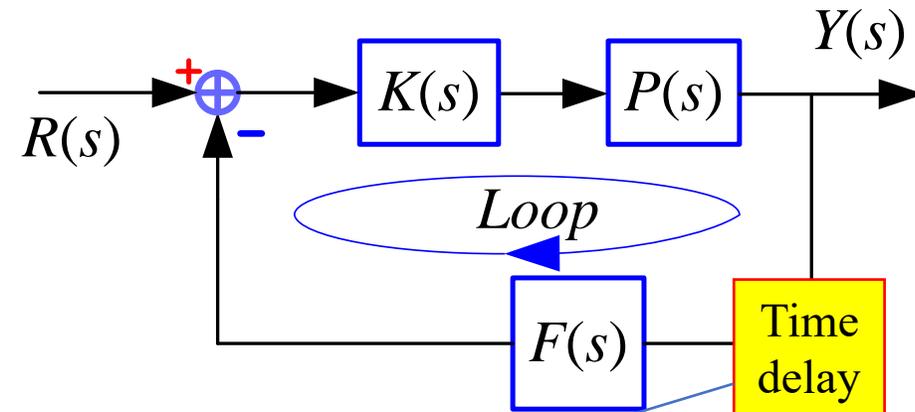


Example (pole-zero map)



Stability Criteria (bode plot)

- It is easy to solve the linear equation, but can not answer questions like “if I increase the gain in $K(s)$, what would happen? The FB system is still stable or not? If not, why it becomes unstable? ”.
- Furthermore, in some system, the characteristic equation is not like a polynomial, thus it is difficult find out the poles directly (such as system with time delay).

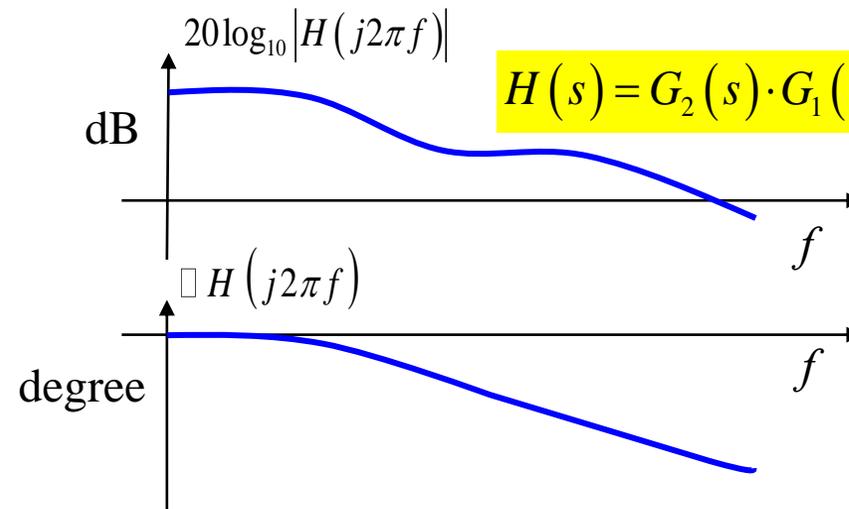
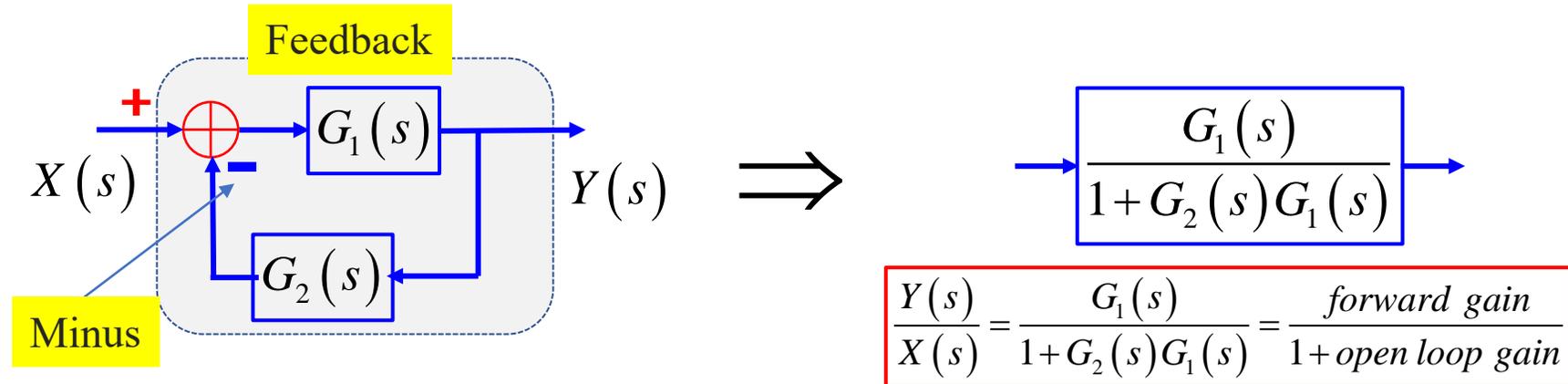


$$\text{NOT like } H_{\text{delay}}(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + b_{m-2} s^{m-2} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0}$$

Stability Criteria (bode plot)

- Still remember the bode plot? It is a very popular method to judge the system stability and also analyze the system performance by its open loop TF.

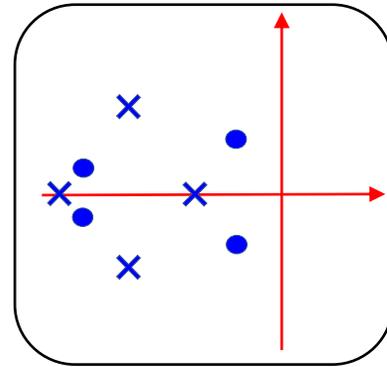
$$H(s)\Big|_{s=j\omega} = H(j\omega) = |H(j\omega)|e^{j\angle H(j\omega)} = |H(j2\pi f)|e^{j\angle H(j2\pi f)}$$



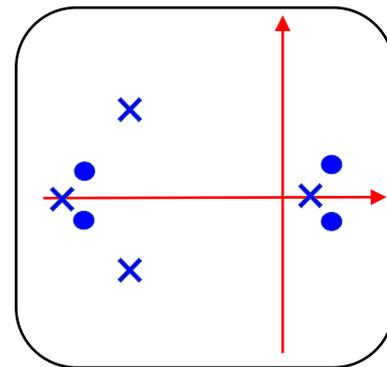
**IMPORTANT:
open loop TF**

Stability Criteria (bode plot)

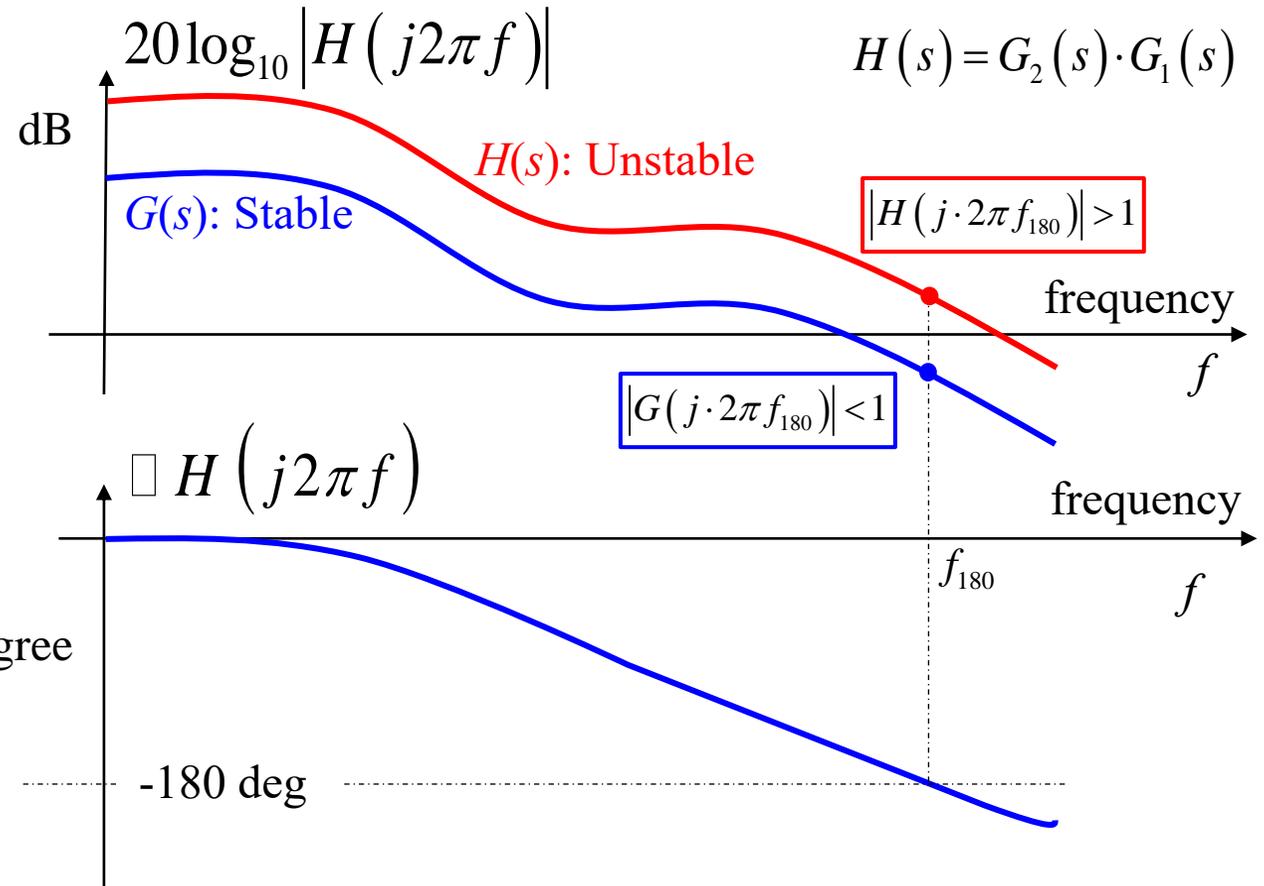
- Suitable for bode plot:
 - 1) Minimum phase,
 - 2) SISO system.



minimum phase

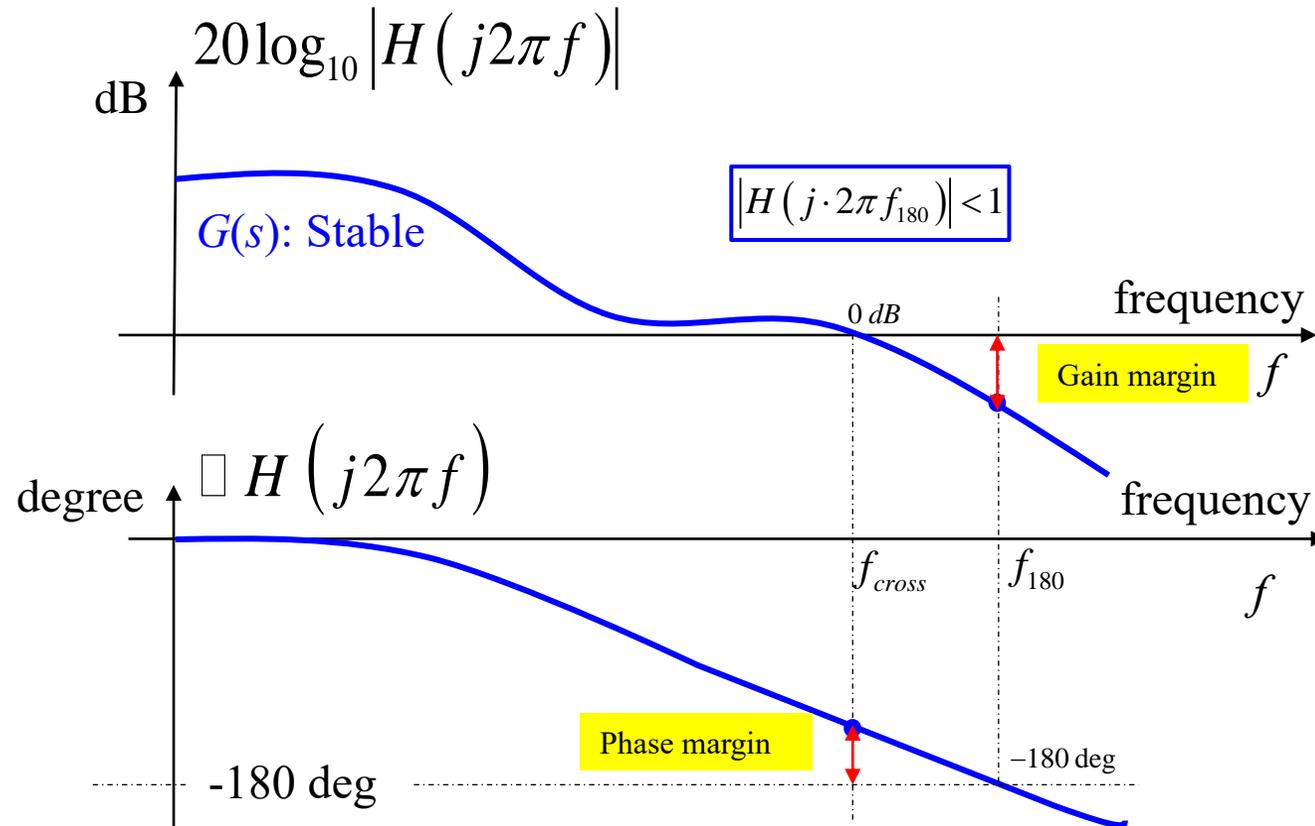


Non-minimum phase



The closed loop is stable, if the open loop gain is less than 1 (0 dB) at a phase of the open loop of -180 degree (or +180, -540, etc.).

Gain Margin and Phase Margin



Gain Margin: $0 - 20\log_{10}|G(j \cdot 2\pi f_{180})|$

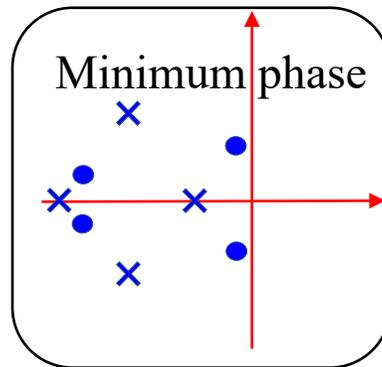
Phase Margin $180 + \angle G(j \cdot 2\pi f_{cross})$

Larger margin → Better robustness

Stability Criteria (Nyquist diagram)

- Benefits of Nyquist diagram

- More information
- Non-minimum phase okay
- MIMO also okay



$$P_{cl} = P_{ol} - N,$$

P_{cl} = number of the close loop RHP poles

P_{ol} = number of the open loop RHP poles

N = times to encircle the $(-1 + 0i)$

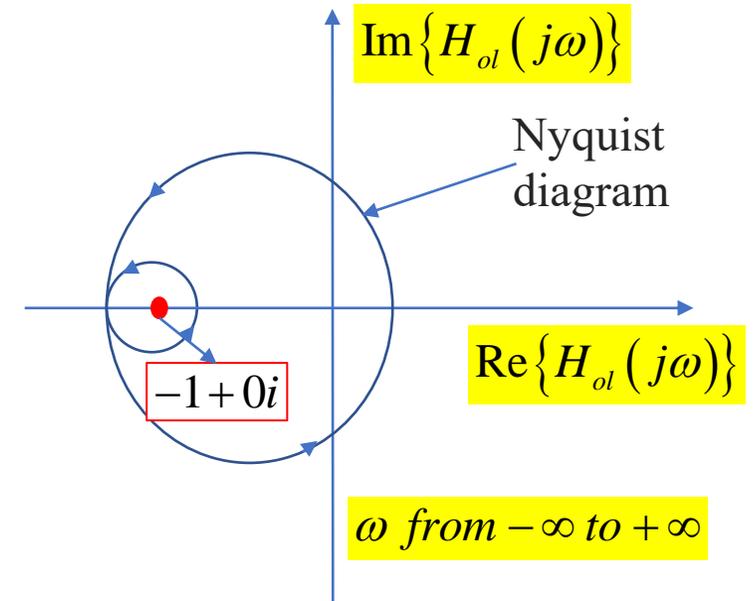
Clockwise: -1

Anti-clockwise: +1

$$H_{OL}(s) = \frac{0.9(1+0.4s)(1+2.5s)}{(s^2+s+1)(1+1.3s)(1.2s-1)}$$

Bode $H_{OL}(j\omega) = |H_{OL}(j\omega)| e^{j\angle H_{OL}(j\omega)}$

Nyquist $H_{OL}(j\omega) = \text{Re}(H_{OL}(j\omega)) + j \cdot \text{Im}(H_{OL}(j\omega))$

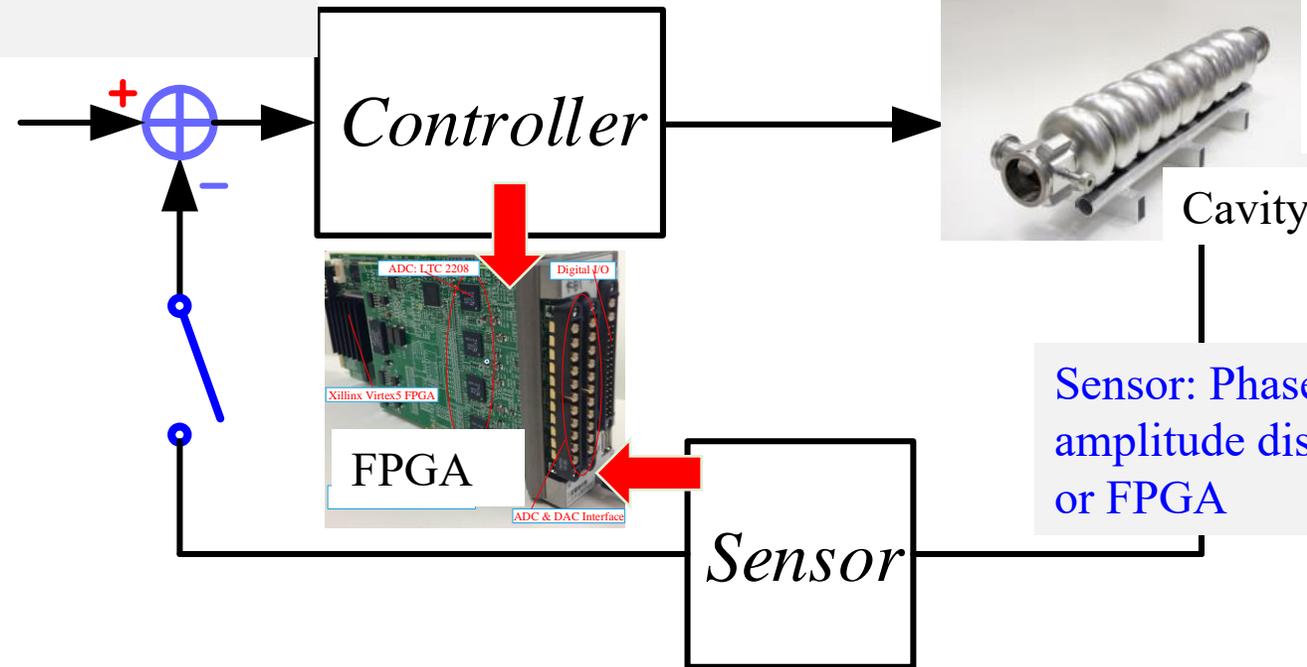


LLRF system

Controller: Electrical control phase shifter or attenuator, FPGA

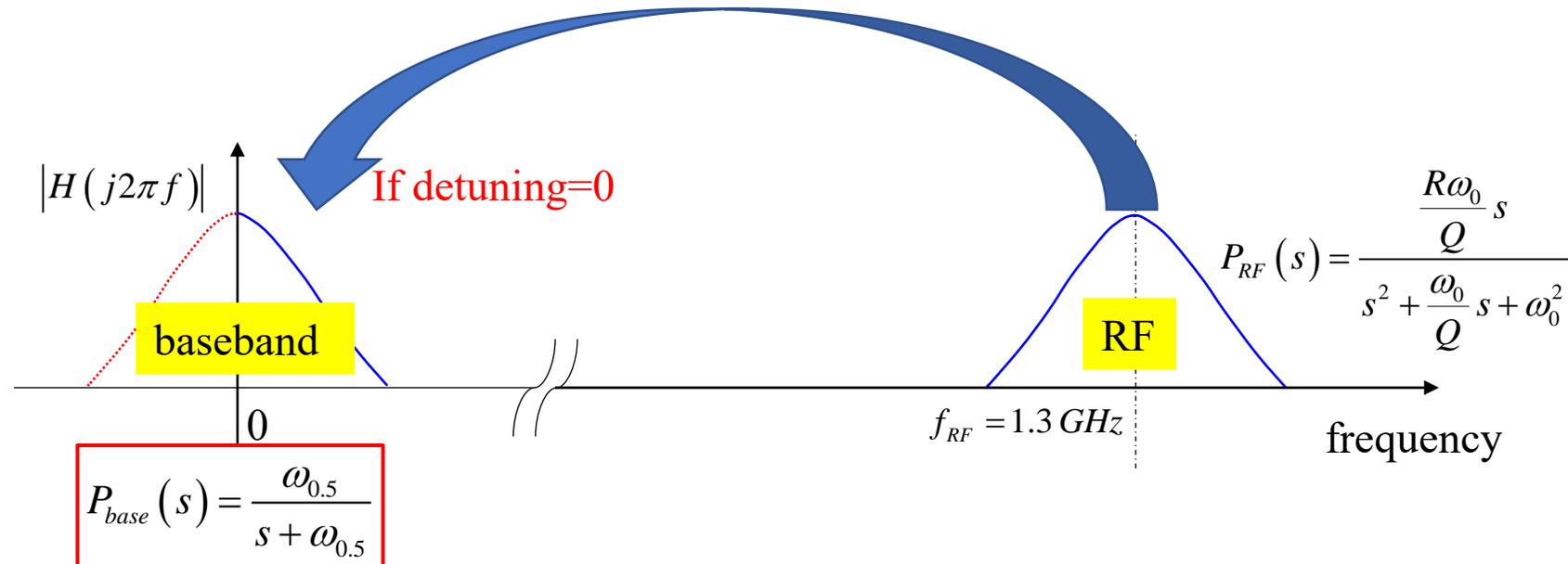
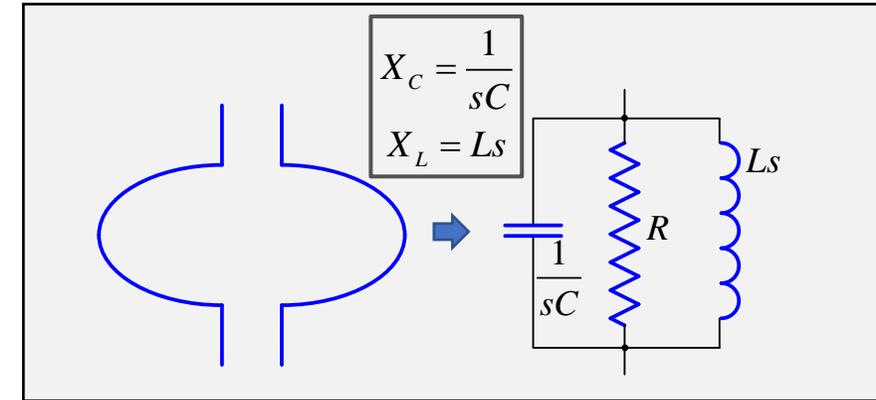
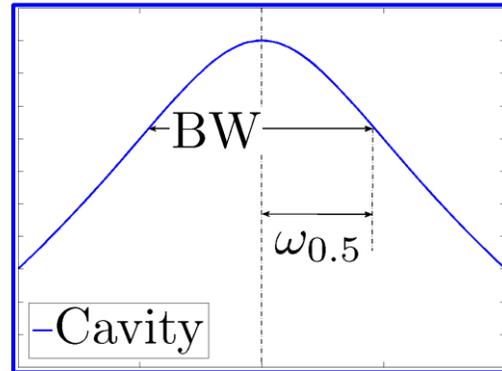
Plant: Cavities, power source, RF Gun, antenna,...

Target: Stabilize the field inside the cavity



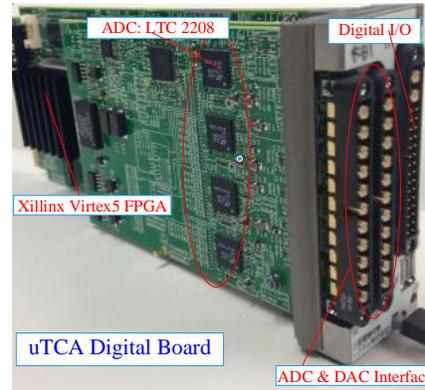
Cavity (detuning=0)

- Cavity is like a parallel resonance circuit.
- First of all, we consider the simplest case: no cross component (detuning=0)



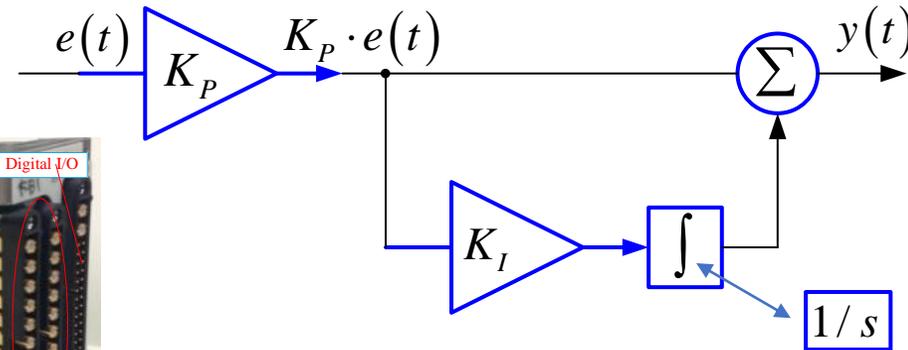
PI controller

- PI control is very popular in the FB control system (& LLRF FB control system)



Usually performed in the FPGA (or DSP)

PI Controller



$$y(t) = K_P \cdot e(t) + K_P \cdot K_I \cdot \int e(t) dt$$

$$Y(s) = K_P \cdot E(s) + K_P \cdot K_I \cdot \frac{E(s)}{s}$$

$$K(s) = \frac{Y(s)}{E(s)} = K_P \left(1 + \frac{K_I}{s} \right)$$

Lapalce Transform

$$\int f(t) dt \Leftrightarrow \frac{F(s)}{s}$$

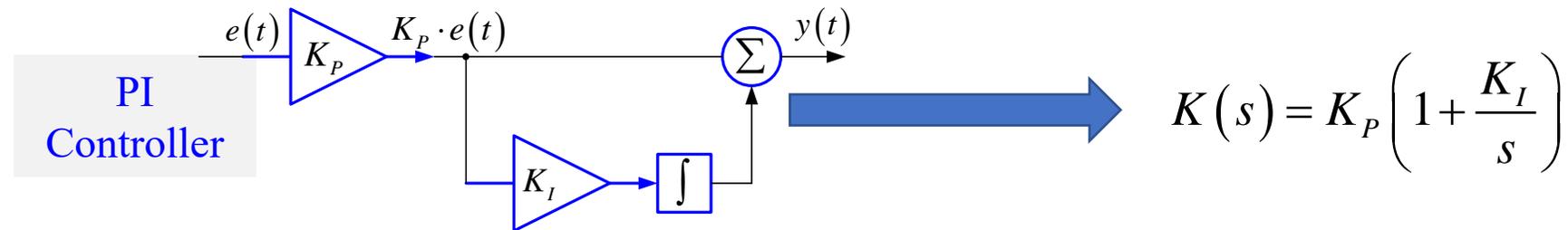
Transfer function

Analytical Study (components)

- PI control is very popular in the FB control system (& LLRF FB control system)
- Cavity and detector are a low-pass filters with different bandwidth.

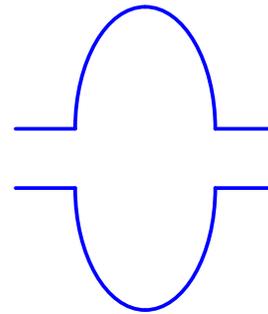
Sys. Components

Transfer function

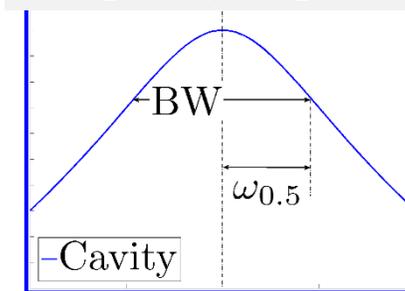


$$K(s) = K_P \left(1 + \frac{K_I}{s} \right)$$

Cavity



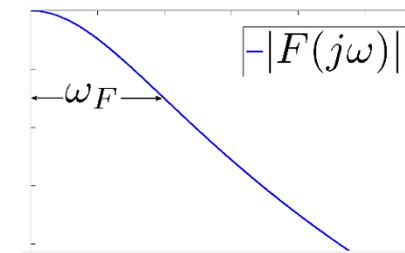
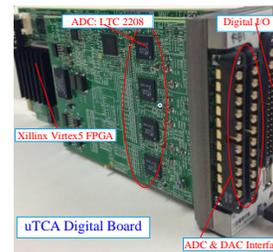
Frequency response



$$P(s) = \frac{\omega_{0.5}}{s + \omega_{0.5}}$$

Half bandwidth

Detector



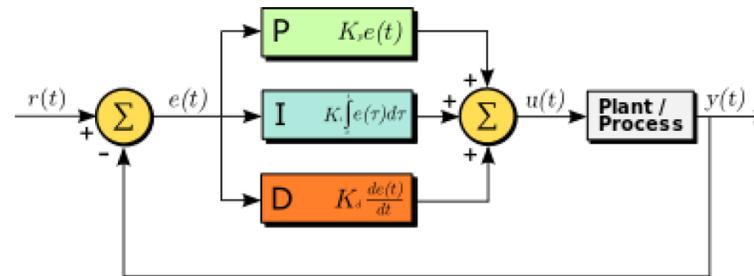
Frequency response

$$F(s) = \frac{\omega_F}{s + \omega_F}$$

Further Types of Feedback Controller

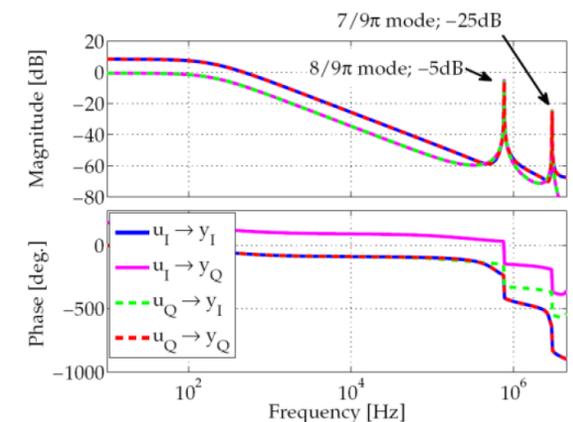
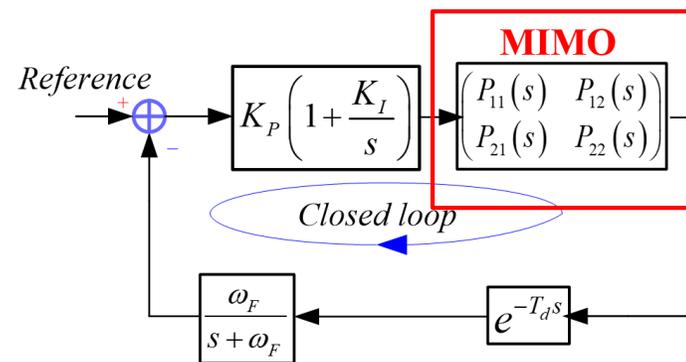
- Classic feedback controller

- P: proportional controller output scales with the input error
- I: integral controller minimizes the steady state error left from the proportional controller correction
- D: differential controller tries to minimize rapid error changes



- Modern feedback controller

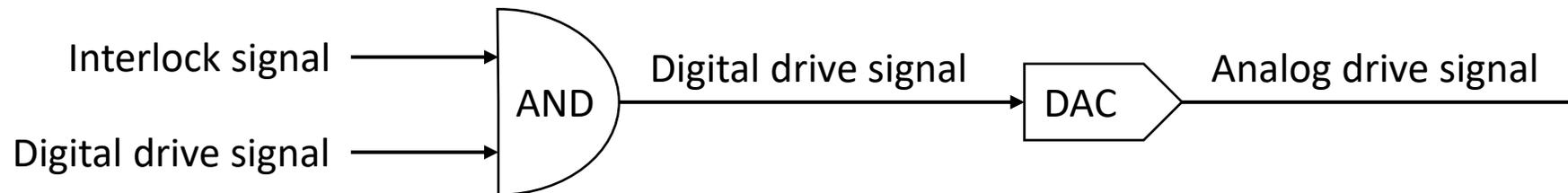
- E.g. 2x2 MIMO (multiple input multiple output) controller (can do PID and more)
 - Cancellation of a passband mode
 - Cancellation of cross coupling between inputs



Example Features of an LLRF System

Interlock

- Every facility typically has a Personal Protection System (PPS) and most facilities have a Machine Protection System (MPS)
- Since the LLRF system is a sub-system of a facility, it must have interlock capabilities
- Typically hardwired in hardware or firmware
 - E.g., logical 'and' just before the DAC

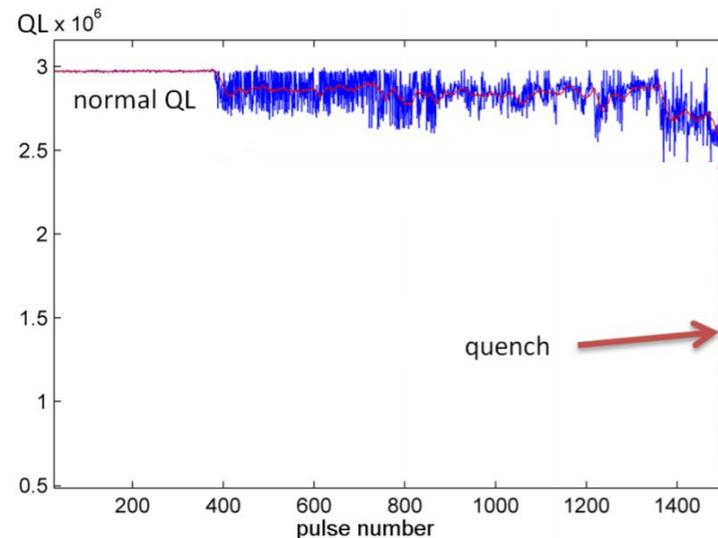
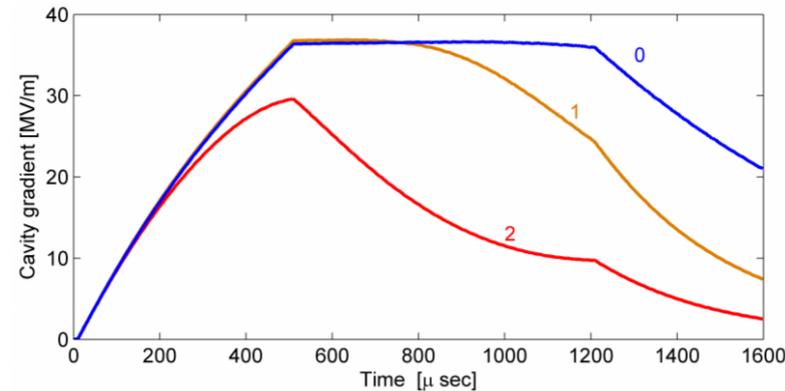
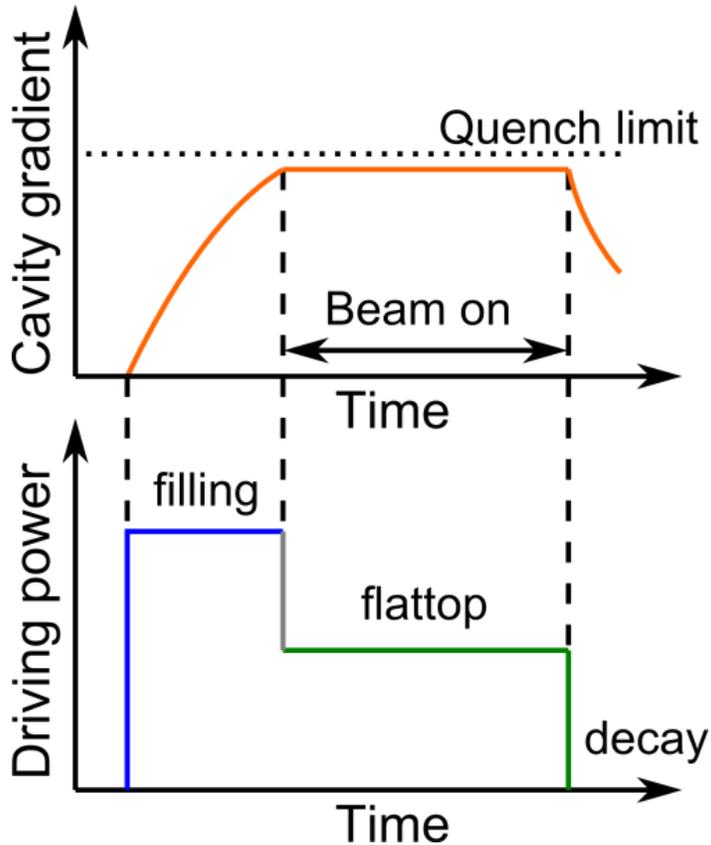


Exception Prevention and Handling



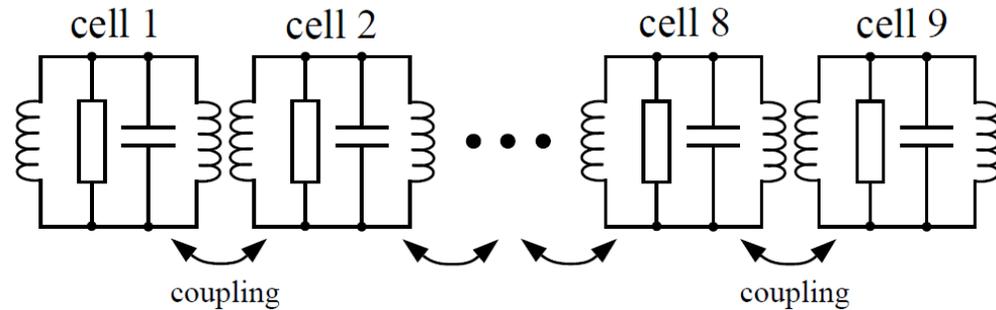
- The LLRF system should prevent certain exceptions
 - Limiters
 - Maximal setpoint voltage
 - Maximal drive signal amplitude
 - Etc.
- The LLRF system should also include a certain degree of exception handling
 - Algorithms for monitoring or computing parameters and for reacting accordingly
 - Turn off RF drive in case of klystron trip
 - Quench detection
 - Etc.

Operation Close to the Quench Limit

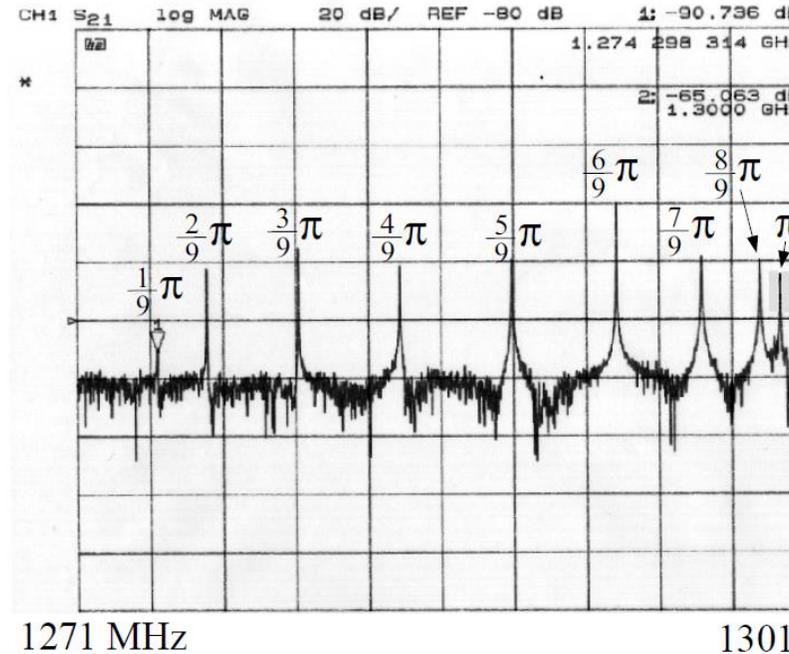


- Quench detection is a common feature of LLRF systems
- If Q_L drops below a predefined limit, the drive is turned off
- Should create interlock for the beam
- RF is turned back on manually or by an automation algorithm

Suppression of Unwanted Passband Modes

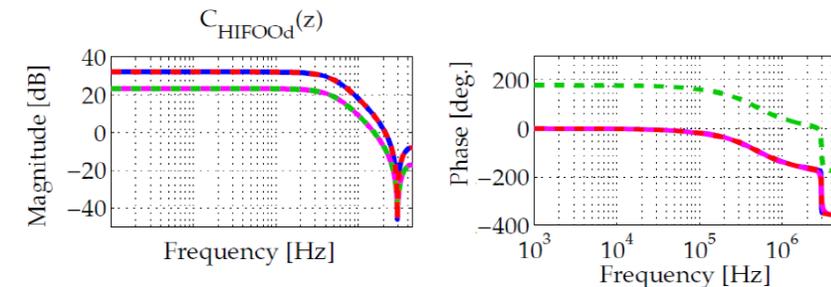


Passband Modes of 9-cell Cavities



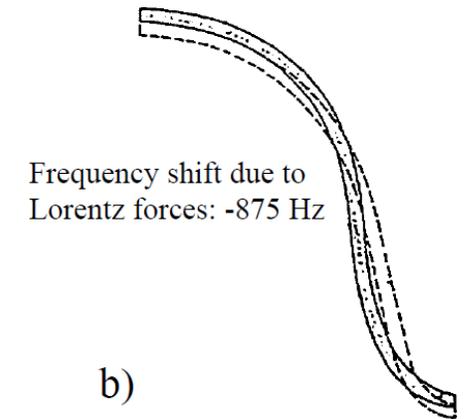
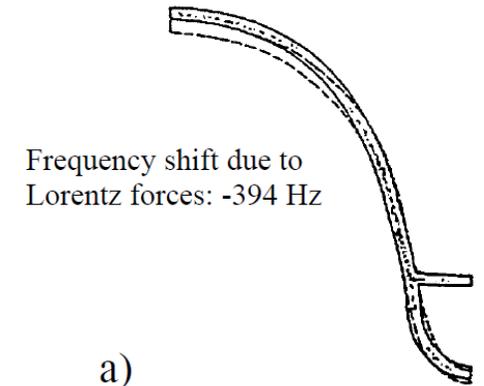
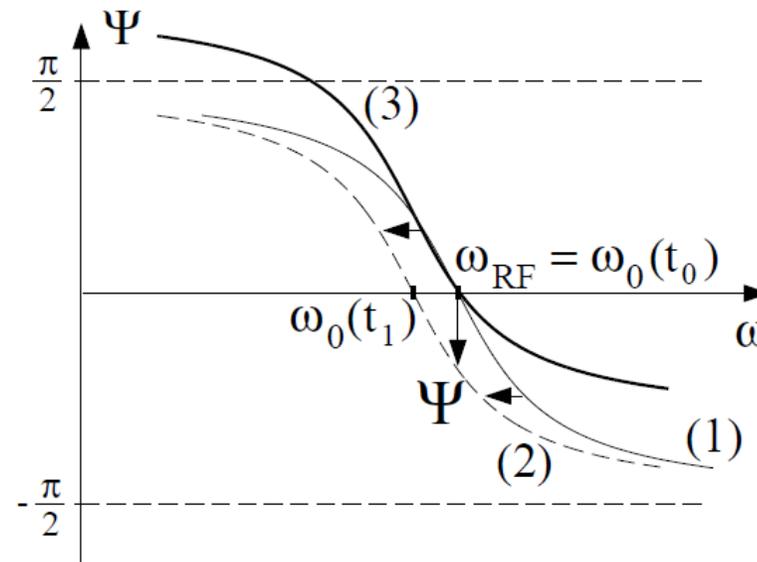
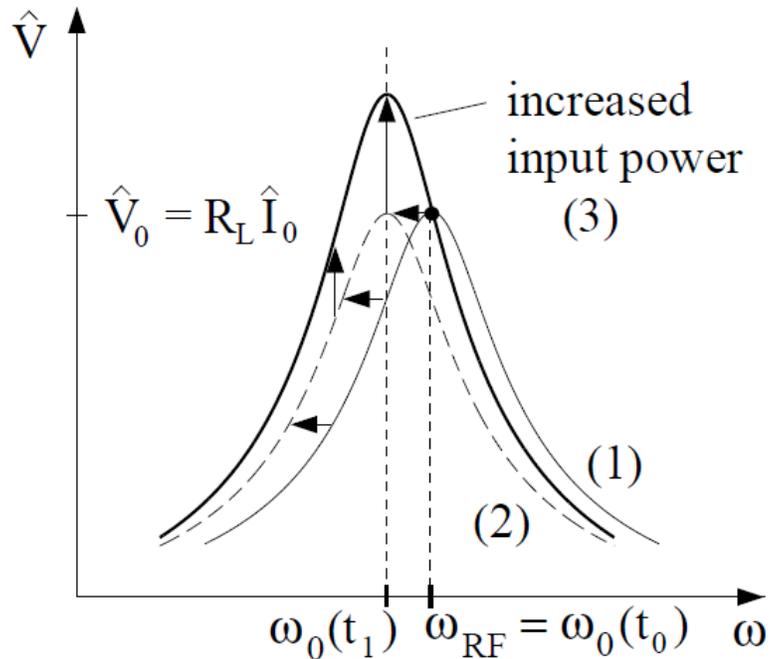
- $f_{\pi} = 1300.091$ MHz
- $f_{8/9\pi} = 1299.260$ MHz
- $f_{7/9\pi} = 1296.861$ MHz
- $f_{6/9\pi} = 1293.345$ MHz
- $f_{5/9\pi} = 1289.022$ MHz
- $f_{4/9\pi} = 1284.409$ MHz
- $f_{3/9\pi} = 1280.206$ MHz
- $f_{2/9\pi} = 1276.435$ MHz
- $f_{1/9\pi} = 1274.387$ MHz

- Implement filter (e.g. Notch filter at ADC) in order to suppress frequency of the $8\pi/9$ -mode



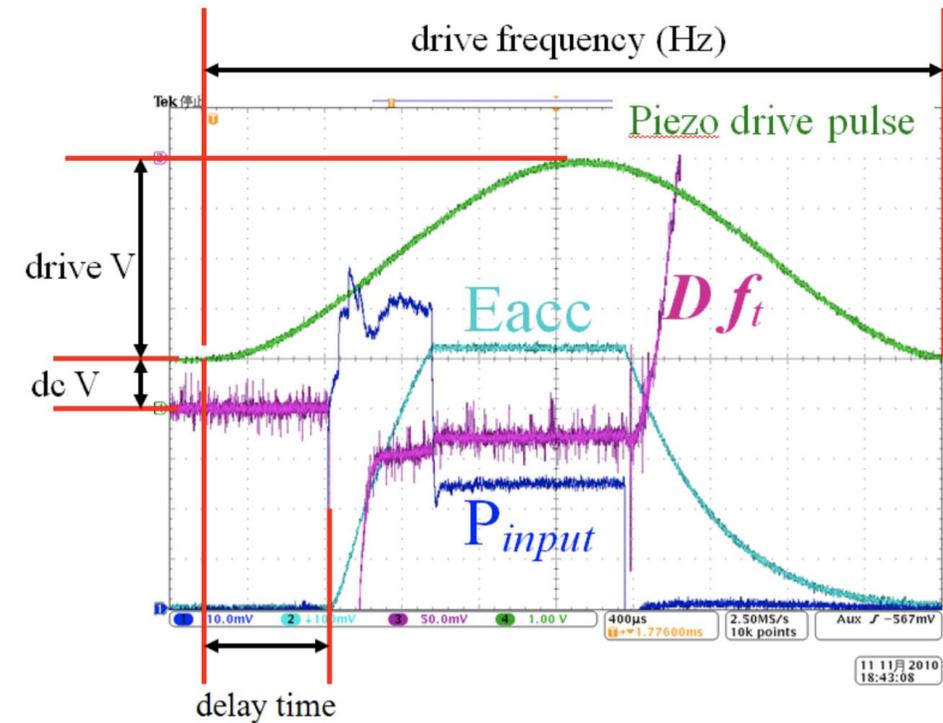
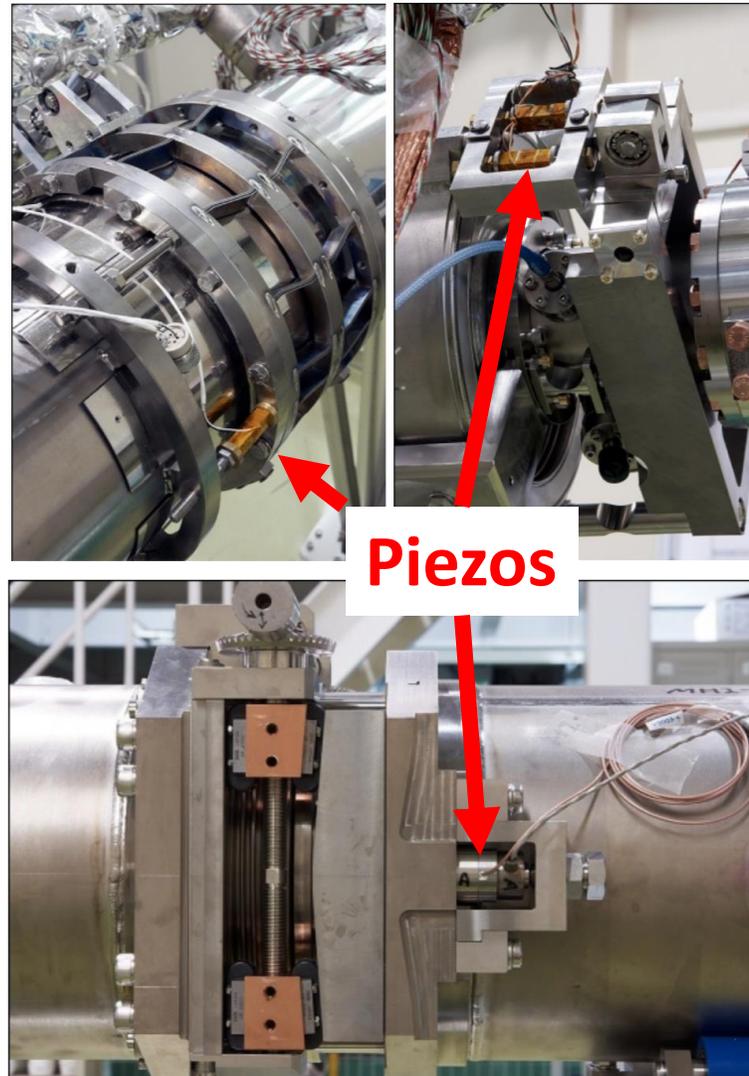
Detuning

- Detuning lowers the amplitude / requires more power to reach the same amplitude
- Detuning induces change of phase
- The sources are Lorentz force detuning and microphonics



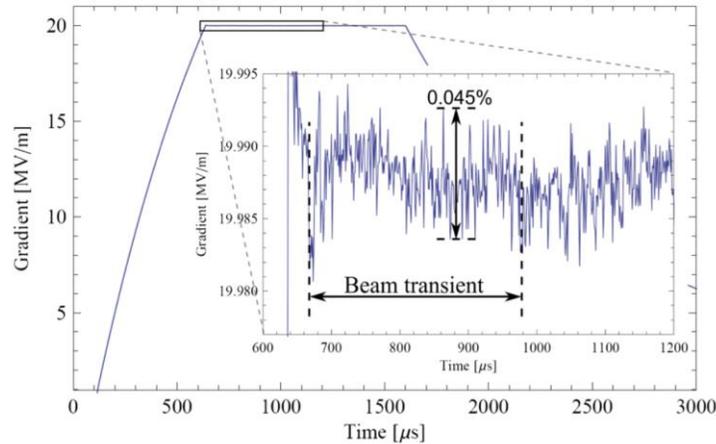
Detuning Compensation

- Motor tuner
 - Slow
 - Pre-tune cavity
 - Compensation of static detuning
- Piezo tuner
 - Fast
 - Compensation of dynamic detuning (E.g. Lorentz force detuning, etc.)
 - Piezo control is typically part of the LLRF system

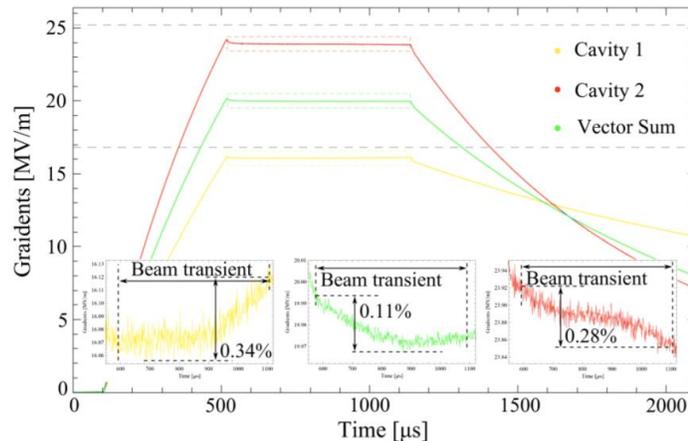


Benchmarking the System Performance

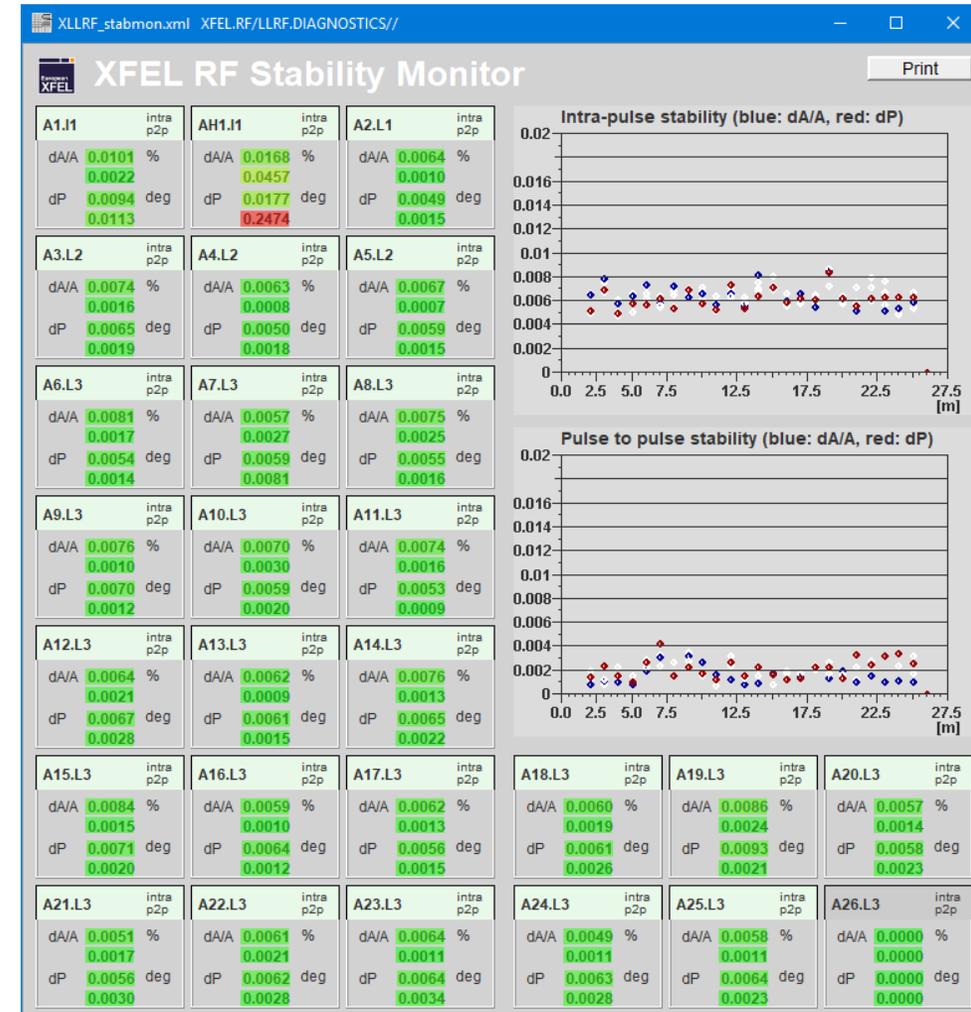
- RF stability (VS)
 - Intra train
 - Inter train
- Long term drifts
- Must be better than requirements for (beam) operation



KEK STF: $Q_L = 2E7$



KEK STF: $Q_{L,cav1} = 9E6$, $Q_{L,cav2} = 3E6$



European XFEL: overview of VS stabilities, requirements: $\Delta A/A \leq 0.01\%$, $\Delta \Phi \leq 0.01$ deg.

Summary and Bibliography

Summary



- What you should learn about, when are planning to get involved with LLRF
 - Your facility
 - What are the requirements? (e.g. for short time and long-time stability, etc.)
 - How to integrate the LLRF system (e.g. interlock, communication, etc.)
 - Theory
 - Cavity
 - RF
 - Signal processing
 - Controller
 - Analog hardware
 - Digital hardware
 - Firmware
 - Software
 - E.g. communication, computations, automation, data analysis, data storage, data visualization, user interface, etc.

Thank you very much for your attention! Questions?



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