Impedance and Beam Instabilities インピーダンスとビーム不安定性

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高エネルギー加速器セミナーOHO'25

Sep 3, 2025, 2:30 PM

3号館セミナーホール

Table of Contents 内容

はじめに

概要

- 私のインピーダンスおよび不安定性に関する知識は限られています。
- たとえば、以下のような研究成果はあります：
  O. Tanaka et al., “Impedance Evaluation of the PF In-Vacuum Undulator: Theory, Simulations, and Measurements,”　Journal of Physics: Conference Series, 1067, 062008 (2018).
- しかし、本テーマは非常に幅広くのため、今日の話は光源加速器周りの話になります。

感謝の言葉

- 中村則雄先生、山本尚人先生からの有益な助言に深く感謝いたします。

Educational Milestones (OHO Seminars)

セミナー開催年	講師名	講義テーマ
1996	Y.H. Chin	ウェイク場、インピーダンスと不安定性
1999	Y.H. Chin	インピーダンスとウェイク場
2010	Y. Shobuda	大強度陽子リングのビーム力学3: ビームの受けるインピーダンスとビームの不安定性の理論
2011	Y. Shobuda	インピーダンス、ウェイク場とロスファクター

http://accwww2.kek.jp/oho/OHOtxt2.html
http://accwww2.kek.jp/oho/OHOtxt4.html

Core Concepts

コアコンセプト

The Bathtub Analogy

- Imagine a toy boat pulled through a bathtub.
- It creates waves that travel and bounce off the walls.
- The waves can affect other boats that follow.
- In an accelerator, the "boat" is a particle bunch, the "tub" is the vacuum chamber, and the "waves" are electromagnetic fields.
- This interaction is called **impedance**.

Video created with Veo 3

What is Impedance?

A Formal Definition

- In a particle accelerator, a "bunch" of particles is the "boat."
- The vacuum chamber and its components are the "bathtub walls."
- The "wake" is an electromagnetic field left behind by the bunch.

The beam coupling impedance quantifies how the accelerator structure responds to beam current oscillations at different frequencies. It determines how much voltage is induced by a given current modulation.

Key Equation:

$$V(\omega)=Z(\omega)I(\omega)$$

- $V(\omega)$: The voltage induced on the beam.
- $Z(\omega)$: The impedance.
- $I(\omega)$: The beam current.

Think of it as: Impedance is the “resistance”（抵抗 テイコウ）of the accelerator to the particle beam, measured at different frequencies.

The Duality of the "Wake" - Time Domain

Two Views on the Same Physics

- **Wake Fields (The Time Domain):** This is the direct, intuitive view, like watching the waves travel behind your boat.
- A "source particle" at the front creates the wake.
- A "test particle" following behind feels the force of that wake.
- This approach focuses on how the force changes over time and distance.

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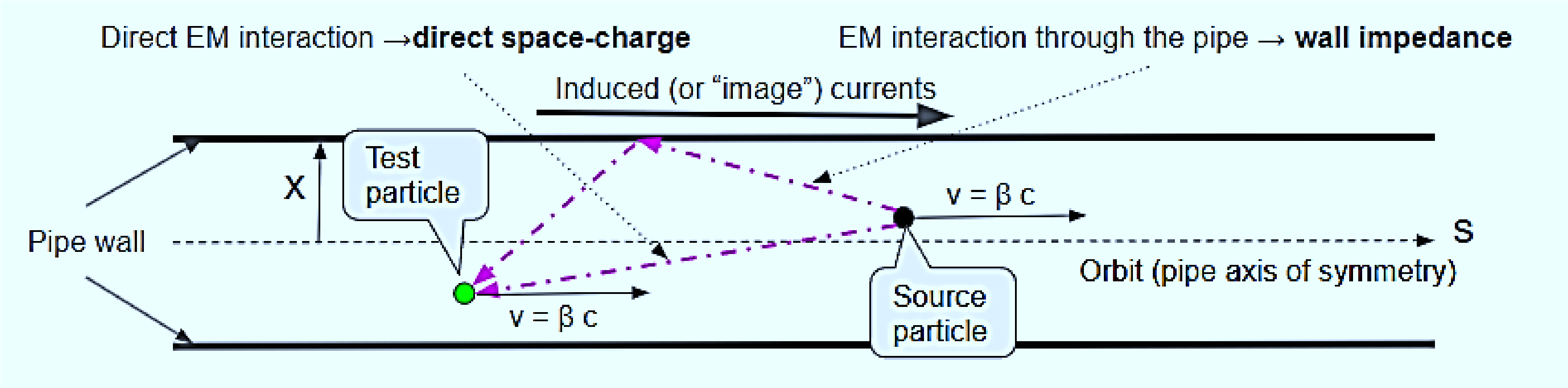
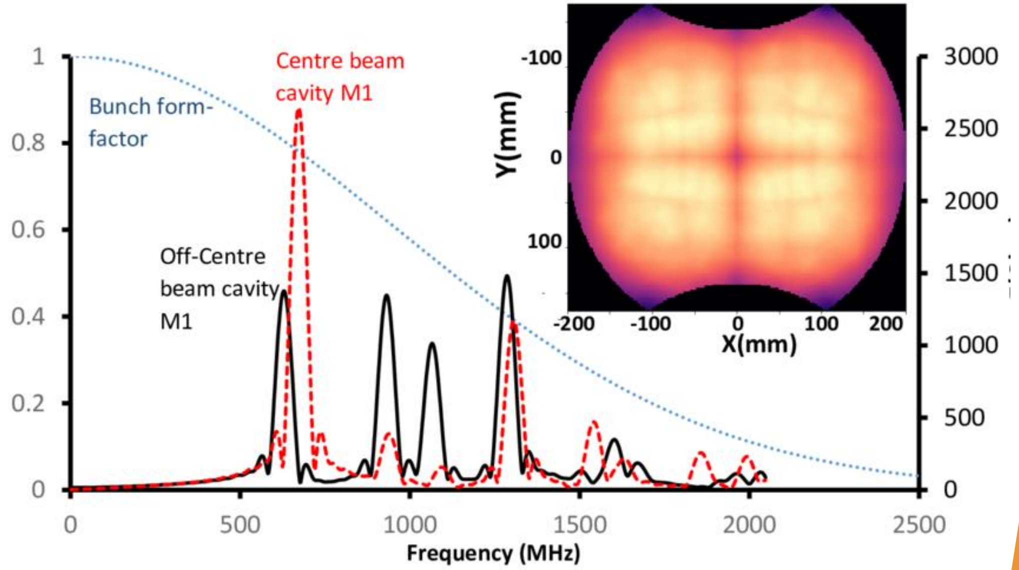


The Duality of the "Wake" - Frequency Domain

Two Views on the Same Physics

- **Impedance (The Frequency Domain):** This is a more abstract, but powerful, way to analyze the effect.
- It's like listening to the sound the water makes as it sloshes back and forth.
- Impedance is a mathematical tool that breaks down the "sloshing" into its specific frequency components.
- This is essential because different parts of the accelerator resonate（共振する） at different frequencies, like a tuning fork.

https://upload.wikimedia.org/wikipedia/commons/5/50/Beach_sea_wave.gif

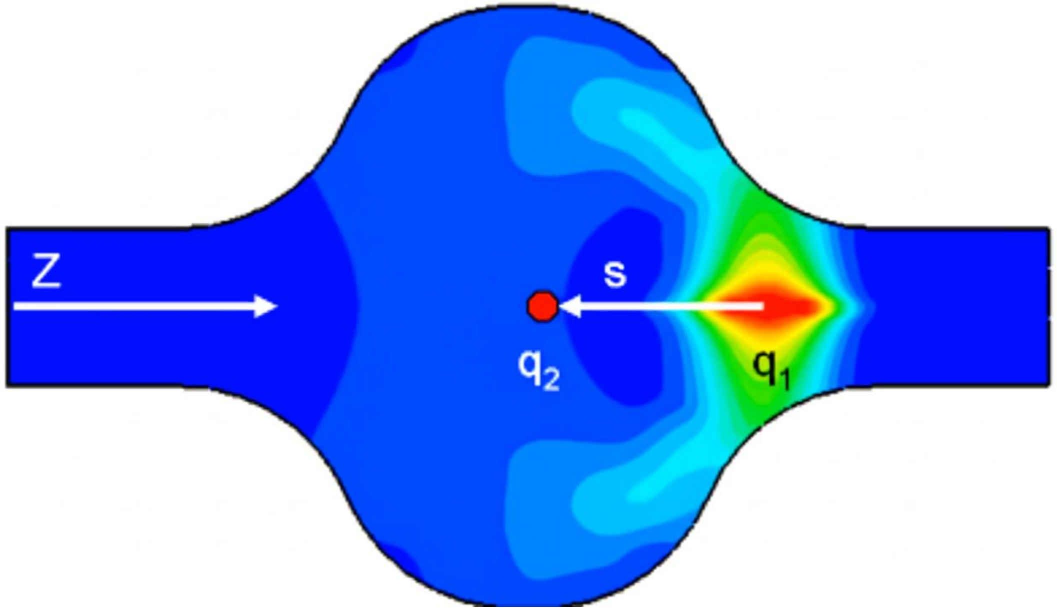


I. Konoplev, et.al. *Beam impedance minimization for accelerator beamline insertion devices*. Review of Scientific Instruments. 91. 074711. 10.1063/5.0007449. 2020.

The Wake Field

Fundamental Concept（基本概念）

- The wake field is the electromagnetic field generated by a charged particle as it moves through an accelerator structure.
- It acts back on other particles that follow it along the beam path.
- **Causality (因果関係):** The wake field cannot act backward in time, so only trailing particles are influenced (W(s)=0 for s < 0).
- **Short-range (短距離) vs. Long-range (長距離):** Wakes can decay quickly (short-range) or persist over many revolutions (long-range).



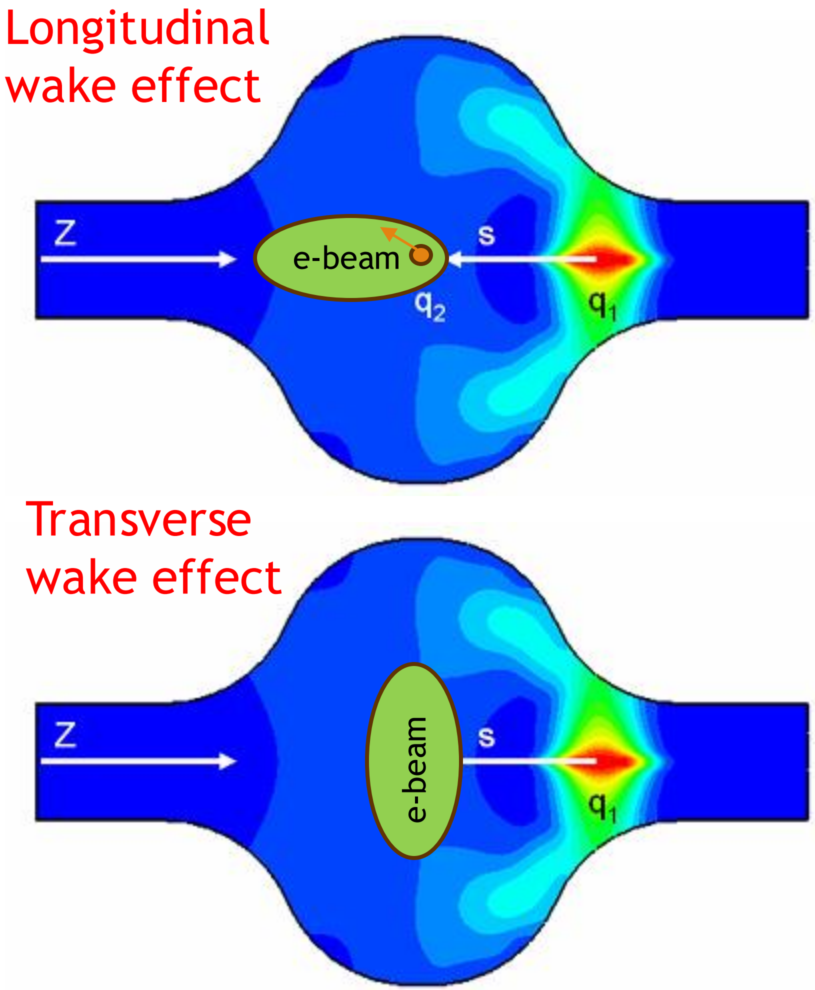
Video created with Veo 3

By the way…
What's wrong with this video?

Longitudinal and Transverse Wake

Categorizing the Effects

- **Longitudinal Wake Field:** This is the part of the wake that affects a trailing particle's energy, either causing it to lose or gain energy. This can change the particle's speed and position within the bunch.
- **Transverse Wake Field:** This is the part of the wake that pushes a trailing particle sideways, giving it a deflection or focusing effect. This can cause the beam to wobble or oscillate.



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The Wake Function
Longitudinal Wake Function

A Quantitative Description

The wake function, W(s), provides a quantitative description of the effect of a source particle on a test particle located a distance s behind it.

Longitudinal Wake Function:

$$W_{\parallel}(s) = -\frac{1}{q}\int E_z(z,t)dz$$

$$\text{for } s > 0$$

- E_z is the longitudinal electric field.
- *This equation gives the energy change per unit charge of the test particle.*

The Wake Function

Transverse Wake Function

The transverse wake function, $W_\perp(s)$, is *defined as the transverse momentum kick per unit charge and per unit displacement of the source particle.*

$$\Delta p_\perp = q_{\mathrm{trail}}\, q_{\mathrm{source}}\, y_{\mathrm{source}} \int W_\perp(s)\, ds$$

- $\Delta p_\perp$ is the **transverse momentum kick** experienced by the trailing particle. It's the change in the particle's momentum perpendicular to its direction of motion.
- q_{trail} is the **charge of the trailing particle** (the one being affected).
- q_{source} is the **charge of the source particle** (the one creating the wakefield).
- y_{source} is the **transverse displacement** of the source particle from the center of the beam pipe. This term is crucial because the transverse wakefield is a **dipole field**, meaning it only exists if the source particle is off-axis.
- $\int W_\perp(s)ds$ represents the integral of the transverse wake function along the length of the accelerator structure. The wake function itself, $W_\perp(s)$, is a function of the longitudinal separation, s, between the source and trailing particles.

Frequency-Domain Impedance

- By applying a Fourier transform to the wake function, we obtain the **impedance** Z(ω), which describes how the induced voltage responds to beam current oscillations at different frequencies:

$$Z_{\parallel}(\omega) = \int_{0}^{\infty} \mathrm{W}_{\parallel}(\mathrm{s}) e^{\frac{i\omega s}{c}} ds,$$

$$Z_{\perp}(\omega) = \frac{i}{c} \int_{0}^{\infty} \mathrm{W}_{\perp}(\mathrm{s}) e^{\frac{i\omega s}{c}} ds.$$

Physical properties of impedance

1. **Causality and analyticity**: Because wake functions are causal, their Fourier transforms (impedances) are analytic functions in the upper half of the complex plane, satisfying Kramers - Kronig relations.
2. **Symmetry**: For real wake functions, $Z(-\omega)^* = Z(\omega)$.
3. **Panofsky-Wenzel theorem**: This fundamental relation links longitudinal and transverse impedance:

$$\vec{\nabla}_{\perp} Z_{\parallel} = i\omega Z_{\perp}$$

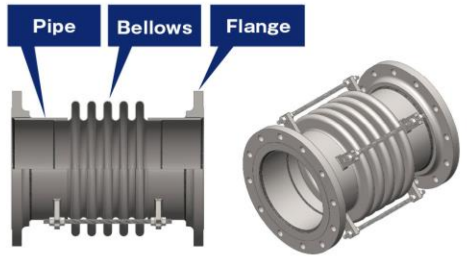
- It states that the transverse kick from a wake field can be directly calculated from the longitudinal field's properties.
- *The Panofsky-Wenzel theorem expresses that the transverse impedance can be derived from spatial derivatives（空間微分）of the longitudinal impedance, under the assumption of slowly varying fields and linear beam dynamics.*

Sources of Impedance

インピーダンスの発生源

Two Major Types of Impedance

Broadband Impedance



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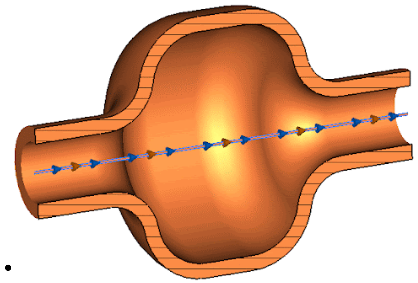
Broadband vs. Narrowband

1. **Broadband Impedance:**
   - Like a low, persistent hum（持続的なハム音）.
   - Comes from small, abrupt changes like bellows or tiny gaps in the pipe.
   - Affects particles **within a single bunch**.

Narrowband Impedance



2. **Narrowband Impedance:**
   - Like a ringing bell（鳴る鐘）.
   - Comes from resonant structures, most notably RF cavities.
   - Can affect a **different bunch** on a subsequent trip around the accelerator.

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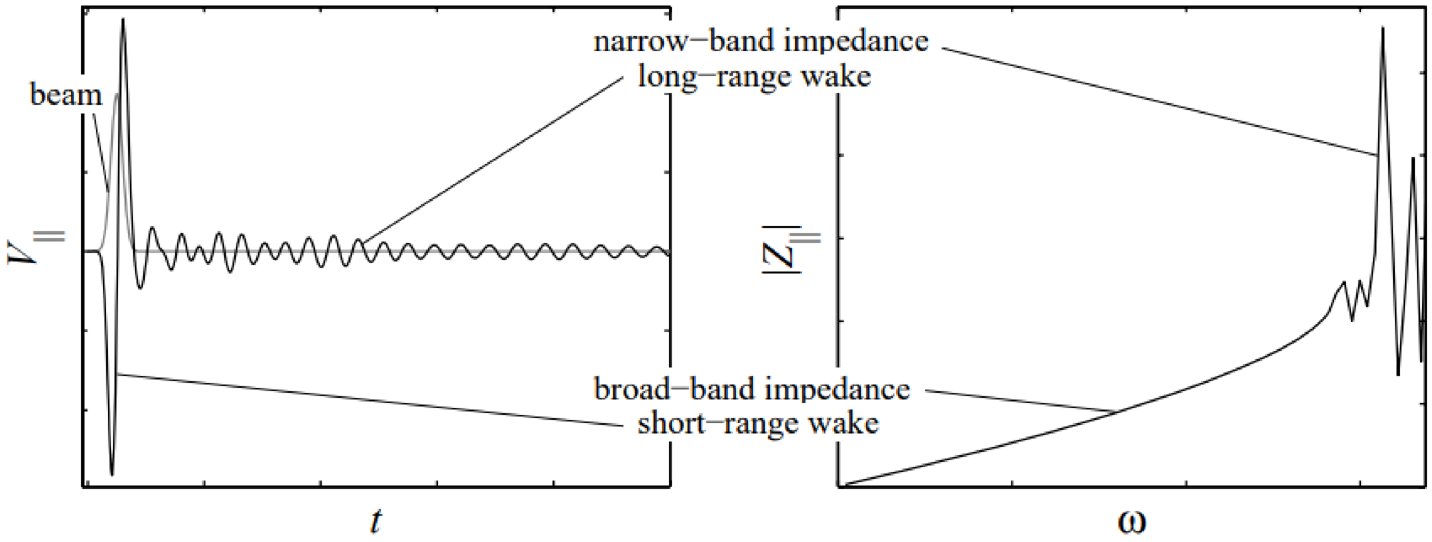
Visualizing Narrowband and Broadband Impedances



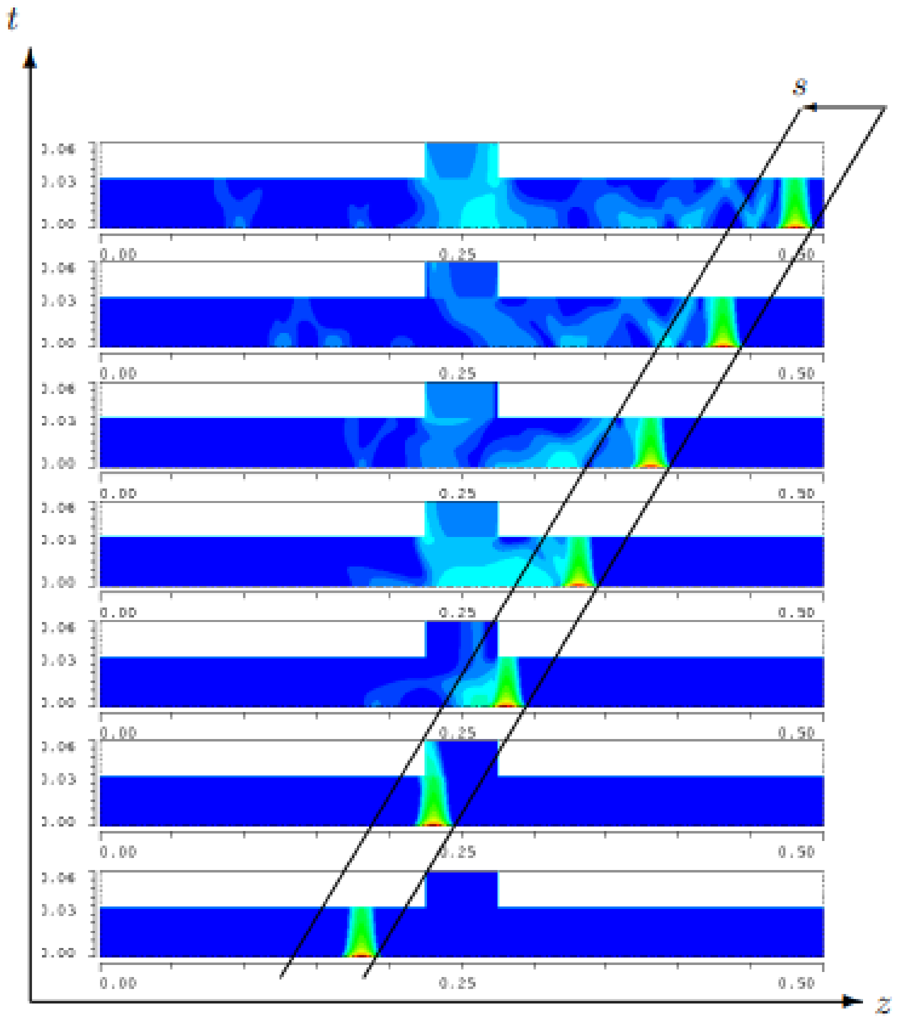
- **Narrowband Impedance Curve:** A corresponding graph should show a very sharp, tall peak in the impedance spectrum at that frequency.
- **Broadband Impedance Curve:** A corresponding graph should show a low, broad, and smooth impedance curve across a wide range of frequencies.

V. Smaluk and R. Wanzenberg, *Geometrical Impedance of the PETRA III Damping Wiggler Section*. ICFA Beam Dynamics Newsletter. 45. 139-146, 2008.

The Major Sources of Impedance
RF Cavities

- RF cavities are a major source of **narrowband impedance.**
- Besides their intended accelerating mode, they can support other resonant frequencies, known as **Higher-Order Modes (HOMs).**
- These HOMs can store energy and "ring" for a long time, leading to interactions with trailing bunches.

Wake fields generated by a Gaussian bunch traversing a cavity

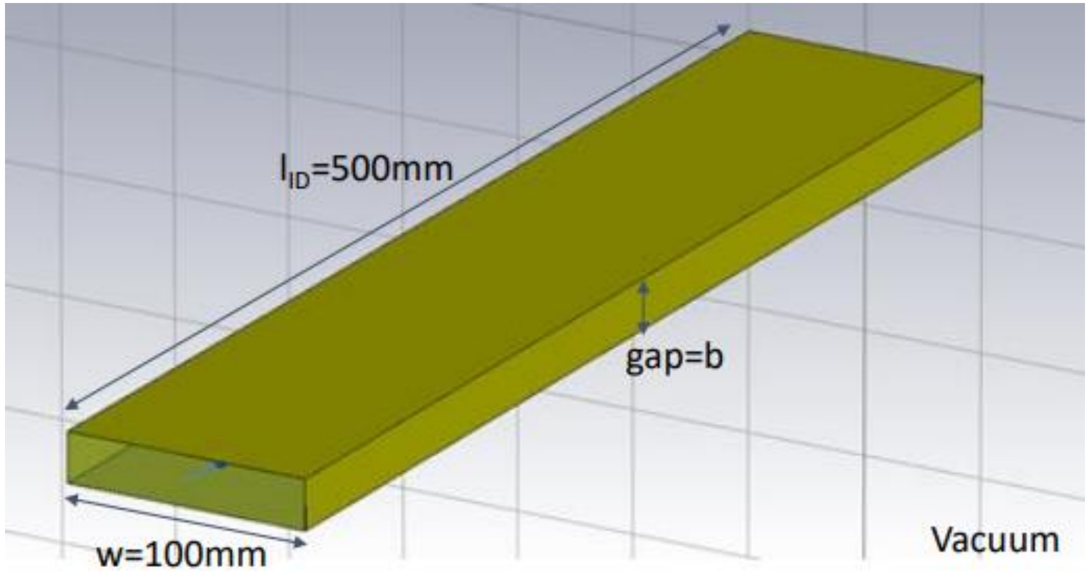


K.Y. Ng, (2006). *Physics of Intensity Dependent Beam Instabilities.* World Scientific.

The Major Sources of Impedance
Resistive-Wall Impedance

- The vacuum chamber walls are not perfect conductors.
- When a bunch passes, it induces a current on the inner surface of the walls.
- Ohmic losses from these currents create lingering electromagnetic fields that act back on the beam.
- This is especially problematic in narrow-aperture chambers, as its strength increases dramatically with a smaller gap.

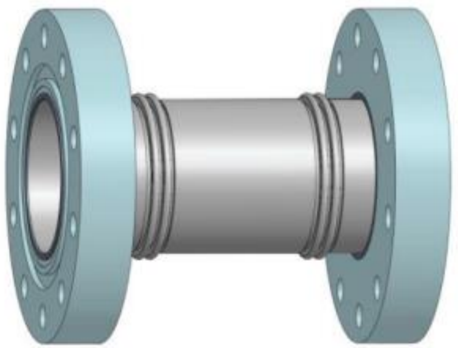
CST model of the undulator RF shielded by the copper sheet.



The Major Sources of Impedance
Geometric Discontinuities

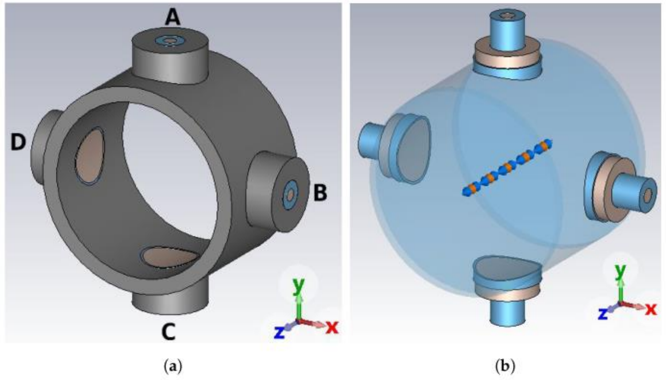
- Abrupt changes in the vacuum chamber, like bellows, flanges, and beam position monitors (BPMs), create localized "wakes" that scatter the beam's electromagnetic fields.
- This impedance is purely due to the shape of the component, not the material, and it's a major source of **broadband impedance.**
- It's a dominant factor for ultra-relativistic beams.

Bellows



Flanges

Beam Position Monitors



1. **G. Wu et al.**, "*Low Impedance Bellows for High-current Beam Operations*", in Proc. IPAC'12, New Orleans, LA, USA, May 2012, paper WEPPC042, pp. 2303-2305.
2. https://share.google/images/Hv7Zdcf3RdEbatwy5
3. https://share.google/images/JnpZMql45GY6WQkJ8

Key Formulas - The RLC Resonator Model

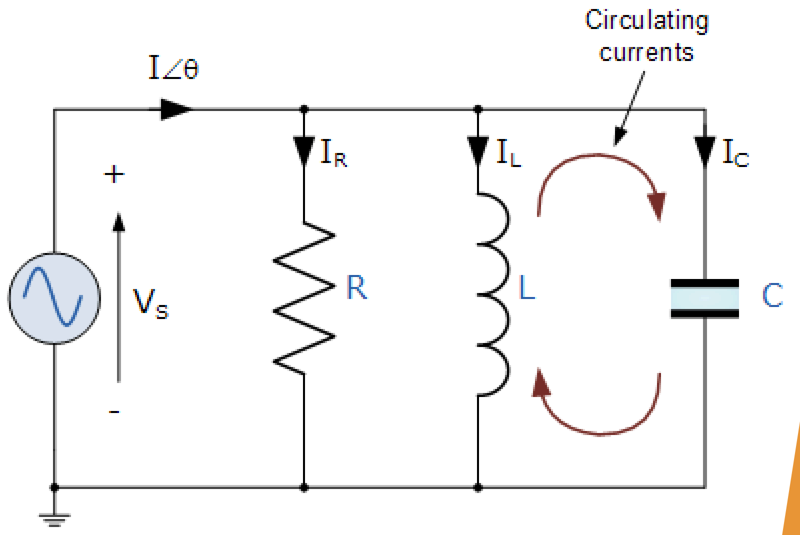
- A resonant mode's impedance can be modeled by an RLC circuit.

Longitudinal Impedance of a Resonant Mode

$$Z_{||}(\omega) = \frac{R_s}{1 + iQ(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega})}$$

- R_s **(Shunt Impedance)**: How effectively a mode converts beam power to voltage.
- **Q (Quality Factor)**: How underdamped the resonance is (how long it "rings").
- ω_r **(Resonant Frequency)**: The frequency at which the mode naturally "rings".

Cavity resonator and equivalent circuit: RLC parallel resonant circuit



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Key Formulas - The Resistive-Wall Impedance

- The transverse resistive-wall impedance per unit length is highly dependent on the beam pipe's half-gap, b.

Transverse Resistive-Wall Impedance

$$Z_{\perp,plates}(\omega) = (i+1)\frac{cZ_0}{2\pi b^3}\sqrt{\frac{1}{2\sigma\omega}}$$

- **$Z_\perp$ (Transverse Impedance)**: The strength of the sideways kick on the beam.
- **b (Half-Gap)**: The vertical distance from the center of the beam pipe to the wall.

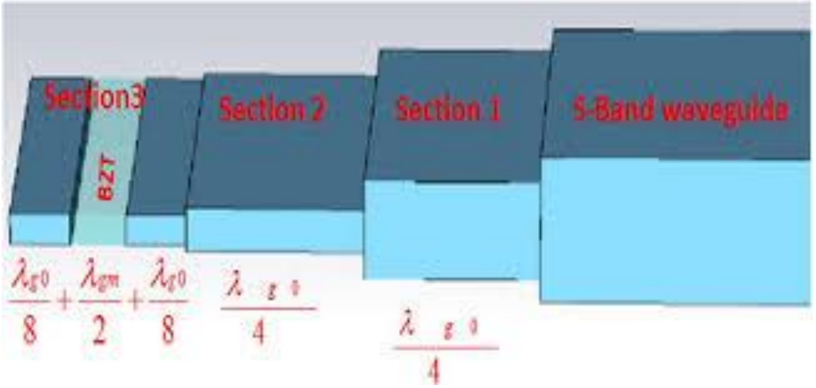
Key takeaway: This formula shows why even a small reduction in the vacuum chamber's vertical size leads to a massive increase in transverse impedance.

Visualizing Geometric Effects

Mitigating Impedance by Design

- **Visual Suggestion 1:** A diagram of a step transition from a wide pipe to a narrow one. This is the "bad" way.
- **Visual Suggestion 2:** A diagram of a gradual, conical taper, which is designed to reduce impedance by smoothing the transition. This is the "good" way.
- **Visual Suggestion 3:** An image of a bellows with RF shielding fingers to show how a discontinuous structure can be made smooth.

Case 1



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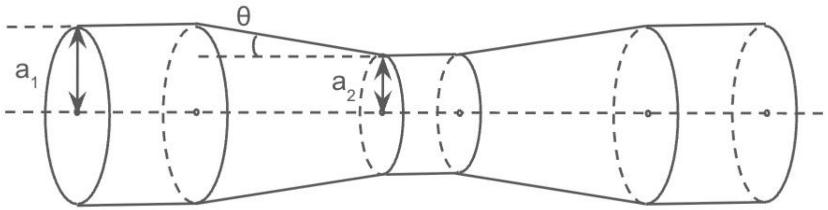
Case 2



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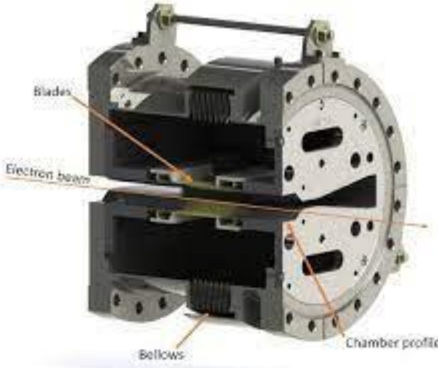
Case 3



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The Unified Nature of Impedance

A Common Challenge

All types of impedance—whether longitudinal or transverse, narrowband or broadband—share a common set of physical effects on the beam.

- **Heating and Energy Loss:** All impedances can cause the beam to lose energy, which must be replenished by the RF system.
- **Bunch Lengthening:** Both longitudinal and transverse impedances can lead to changes in the bunch's shape and length, a phenomenon called potential well distortion.
- **Instability:** The most critical effect is the onset of instability, which can degrade the beam's quality or lead to beam loss.

Cavity Impedance as a Geometrical Effect

- RF cavities, while essential for acceleration, are a major source of impedance due to their complex geometry.
- The wake fields they generate are a form of **geometrical impedance**, which arises from the shape of the component itself, even if it were a perfect conductor.

R. Nagaoka, *Collective Instabilities in Low Emittance Rings*, presented at the Joint Workshop on Future Tau-Charm Factory, LAL, Orsay, France, 4-7 December 2018.

Impedance Budgeting

インピーダンスバジェット

The Impedance Budget

Putting It All Together

- An impedance budget is a systematic evaluation and summation of all the known impedance contributions from every component in a machine.
- This budget is crucial for ensuring that the total beam current can be stored without causing instabilities, and it acts as a strategic roadmap for future upgrades.

Case Study: Photon Factory

The Photon Factory (PF) is a synchrotron radiation facility at KEK in Tsukuba, Japan, that provides high-intensity photon beams for a wide range of scientific and industrial research.

Photon Factory: Sources of Impedance

- **Primary Sources:** The main sources of impedance in the KEK Photon Factory complex are the radiofrequency (RF) cavities, which generate parasitic higher-order modes (HOMs), and modern, small-aperture devices like in-vacuum undulators (IVUs). Other contributors include pumping slots and beam position monitors (BPMs).
- **Design Challenges:** The high beam currents of the PF can lead to strong coupling with the machine's impedance, which can overcome natural damping and cause instabilities. The introduction of IVUs with their low-gap design was a major concern for the total impedance.
- **Mitigation:** Resonant impedance from RF cavities is managed by using HOM dampers to significantly reduce the quality factor of parasitic modes. For IVUs, a rigorous evaluation process using theoretical formulas, simulations, and measurements is used to mitigate their impedance contribution. The transverse "sawtooth" instability is suppressed by using octupole magnets, although a high-gain feedback system was found to unexpectedly exacerbate this instability.

Case Study: BESSY II

BESSY II: Sources of Impedance

Primary Sources: The RF cavities are the largest individual contributor to impedance at BESSY II. Other significant sources include the insertion device chambers, beam position monitors, and discontinuities at the bellows and flanges.

Design Challenges: Components of the storage ring are designed to minimize wakefields, as electron beam instabilities would degrade the quality of the synchrotron radiation. Collective effects are more severe in the single-bunch operational mode compared to the standard multi-bunch mode.

Mitigation: Discontinuities at bellows and flanges are mitigated by shielded, sliding RF fingers. Additionally, four harmonic cavities are used to lengthen the electron bunches, which increases beam lifetime.

S. Khan, *Beam Impedance Study for the BESSY-II Storage Ring*, PAC'97 , paper 7V027.

BESSY II is a third-generation synchrotron radiation source operated by the Helmholtz-Zentrum Berlin in Germany.

By the way…
Why bunch lengthening helps to avoid instabilities??

Case Study: Soleil

SOLEIL is the French synchrotron, both a large scale facility and a research laboratory. Shareholding of the French public centers CNRS and CEA.

Synchrotron SOLEIL: Sources of Impedance

- **Primary Sources:** The main sources of impedance are the RF cavities and the numerous geometric discontinuities in the vacuum chamber, such as shielded bellows, flanges, and beam position monitors (BPMs).
- **Design Challenges:** The small-aperture vacuum chamber, which is necessary for achieving low beam emittance, increases the machine's coupling impedance and its susceptibility to instabilities.
- **Mitigation:** The RF cavities' high impedance is managed by a fast feedback system. Passive harmonic cavities are also used to lengthen the bunch, which helps to mitigate instabilities and intra-beam scattering.

R. Nagaoka, *"Numerical Evaluation of Geometric Impedance for SOLEIL"*, in Proc. EPAC'04, Lucerne, Switzerland, Jul. 2004, paper WEPLT081, pp. 2041-2043.

Case Study: ESRF-EBS

ESRF-EBS: Sources of Impedance

- **Primary Sources:** The main impedance sources are the numerous vacuum chamber components, such as shielded bellows, flanges, tapers, and beam position monitors (BPMs), rather than the RF cavities.
- **Design Challenges:** To achieve a low beam emittance, the ESRF-EBS has a compact Hybrid Multi-Bend Achromat (HMBA) lattice that requires smaller-aperture vacuum chambers, which directly increases the machine's overall impedance.
- **Mitigation:** The facility replaced its RF cavities with new, strongly damped single-cell cavities to push the instability threshold well above the nominal beam current. Additionally, a patented in-house design for RF fingers was developed to electrically shield the bellows, reducing their impedance contribution by a factor of four.

***ESRF-EBS** The European Synchrotron Radiation Facility's Extremely Brilliant Source is the first high-energy fourth-generation synchrotron, with X-ray performances increased by a factor 100 compared to the previous synchrotron. It located in Grenoble, France.*

1. **T. F. Günzel, L. Farvacque, T. Perron, and J.-L. Revol**, "*Evolution of the Machine Impedance following the ESRF Upgrade to Low-Gap NEG Coated Aluminium Chambers*", in Proc. PAC'05, Knoxville, TN, USA, May 2005, paper MPPP021, pp. 1712-1714.
2. **European Synchrotron Radiation Facility**, *New HOM-damped cavities in the storage ring*, ESRF Highlights 2011.
3. **European Synchrotron Radiation Facility**, *Boosting the beam: RF cavities for EBS*, ESRF Extremely Brilliant Source (blog), published 23 June 2017.

Case Study: MAX IV

*MAX IV is an accelerator complex and a synchrotron radiation source in Sweden near the city of Lund. The first of the 4th generation sources, with an emittance of less than 1 nm*rad.*

MAX IV: Sources of Impedance

- **Primary Sources:** The main sources of impedance at MAX IV are the 100 MHz radio-frequency (RF) cavities, particularly their higher-order modes (HOMs), and geometrical components within the vacuum chamber like bellows and flanges. During commissioning, HOMs were identified as the "biggest challenge" in terms of collective effects.
- **Design Challenges:** The Multi-Bend Achromat (MBA) lattice, designed to achieve ultra-low emittance, necessitates a compact structure with small magnet gaps and a low-conductance vacuum chamber. This design choice inherently increases the risk of collective instabilities, such as those driven by resistive wall impedance.
- **Mitigation:** The RF system was designed with a low frequency (100 MHz) to produce long electron bunches, and passively operated third-harmonic Landau cavities are used to further lengthen them, alleviating instabilities. The cavities are capacitively loaded to shift the HOM frequencies away from the bunch spectrum. The 3 GeV ring also utilizes a fully Non-Evaporable Getter (NEG)-coated copper vacuum system, which provides distributed pumping and contributes to stability.

Table 2: Major Impedance Contributors

Facility	RF Cavities (HOMs)	Vacuum Chamber Geometry	Notable Mitigation
Photon Factory (PF)	HOMs were the primary source of longitudinal CBIs in old cavities.	In-vacuum undulator tapers and copper plates are dominant impedance contributors.	The old RF cavities were replaced with newly-developed damped cavities to suppress HOM impedances.
MAX IV	HOM-damped cavities are designed to minimize impedance.	Smooth geometry with NEG coating.	Commissioning required active bunch-by-bunch feedback systems to suppress longitudinal and transverse instabilities.
Soleil	Superconducting, strongly HOM-damped cavities.	Transitions between achromats, resistive-wall from NEG layer.	Use of superconducting cavities with HOM couplers to provide strong attenuation of impedance.
ESRF-EBS	Strongly damped HOMs	Smooth chamber profile optimized for impedance	Smooth chambers and active feedback are standard.
BESSY II	RF cavities and multi-cell resonators	Flat elliptical vacuum chamber	Injection system and RF cavity designs

Instabilities

不安定性

Instabilities:

When the "Wake" Becomes a Problem

- When the energy from these wakes builds up faster than it can naturally dissipate, the beam becomes unstable.
- This can cause the bunches to oscillate or distort, leading to a loss of performance or even beam loss.

Broad Classification: Longitudinal and Transverse.

- **Longitudinal:** Affects particle energy and timing.
- When the beam interacts with longitudinal impedance, it loses energy per turn due to resistive wake fields.
- The **energy loss per turn** *U* due to the longitudinal impedance $Z_{\parallel}$ is given by:

$$U = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\tilde{I}(\omega)\right|^2 Re[Z_{\parallel}(\omega)]\,d\omega.$$

- *This integral represents the total power dissipated by the beam current spectrum $\tilde{I}(\omega)$ in the resistive (real) part of the impedance.*

Instabilities:

When the "Wake" Becomes a Problem

- **Transverse:** Affects the beam's position.
- In the transverse plane, the beam interacts with the **transverse impedance** $Z_\perp$, resulting in tune shifts and possible instabilities.
- The **transverse coherent tune shift** for mode μ:

$$\Delta Q_\mu = \frac{eI_0\beta}{4\pi E_0} \sum_{p=-\infty}^{\infty} \frac{Z_\perp(\omega_0(pM+\mu))}{\omega_0},$$

- where β is the beta function at the location of the impedance.
- *A coherent tune shift is a shift in the oscillation frequency of the beam centroid due to collective forces. It differs from incoherent tune spread caused by nonlinearities or individual particle dynamics.*

Single-Bunch vs. Multi-Bunch Instabilities

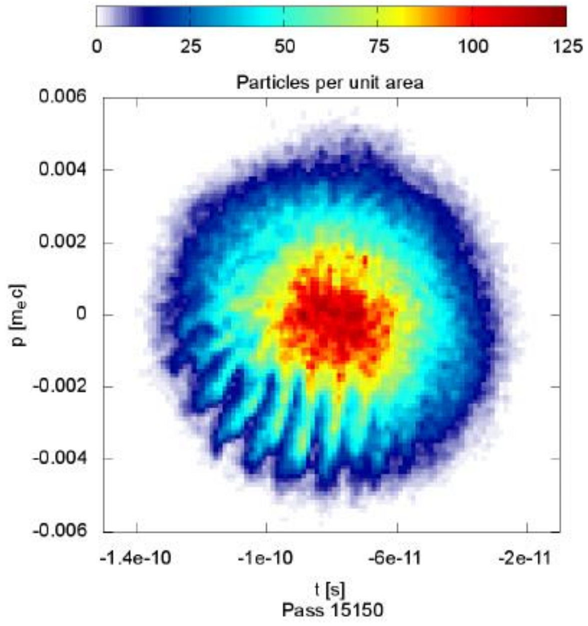
A Different Kind of Interaction

- **Single-Bunch:** Driven by short-range wakes that decay quickly, affecting particles within a single bunch. Examples: Head-Tail, Transverse Mode Coupling Instability (TMCI).
- **Multi-Bunch:** Driven by long-range wakes from high-Q resonant modes that persist and affect subsequent bunches. Example: Coupled-Bunch Instabilities (CBIs).

Microwave Instability

Microwave instability is a collective beam effect in which short-wavelength longitudinal wake fields cause the bunch energy spread and length to grow rapidly once the beam current exceeds a threshold.

- This is a single-bunch effect driven by short-range transverse wake fields.
- **Primary Sources:** Microwave instability (MWI) is a longitudinal instability caused by the broadband impedance of the vacuum chamber, which can deteriorate performance through bunch lengthening and increased energy spread. Sources of this broadband impedance include geometric discontinuities like bellows, flanges, and tapers.
- **Design Challenges:** The drive for low-emittance rings with small vacuum chamber apertures increases the overall impedance, making the beam more susceptible to MWI. MWI's effect on energy spread can be as significant as that of intrabeam scattering (IBS).
- **Mitigation:** A primary strategy to mitigate MWI is bunch lengthening, which reduces the peak particle density. This is achieved using a low main radio-frequency (RF) and passively operated third-harmonic Landau cavities, as exemplified at the MAX IV facility.



Rusanov, Andrii & Chou, Ping & Hsinchu, & Taiwan,. Collective Effects Simulations for the TPS Storage Ring.

The Keil-Schnell Criterion

A Fundamental Stability Limit

This criterion provides a fundamental limit on longitudinal impedance to prevent microwave instability.

$$\left|\frac{Z_{||}}{n}\right| < \frac{2\pi\alpha E\delta^2}{eI_0}$$

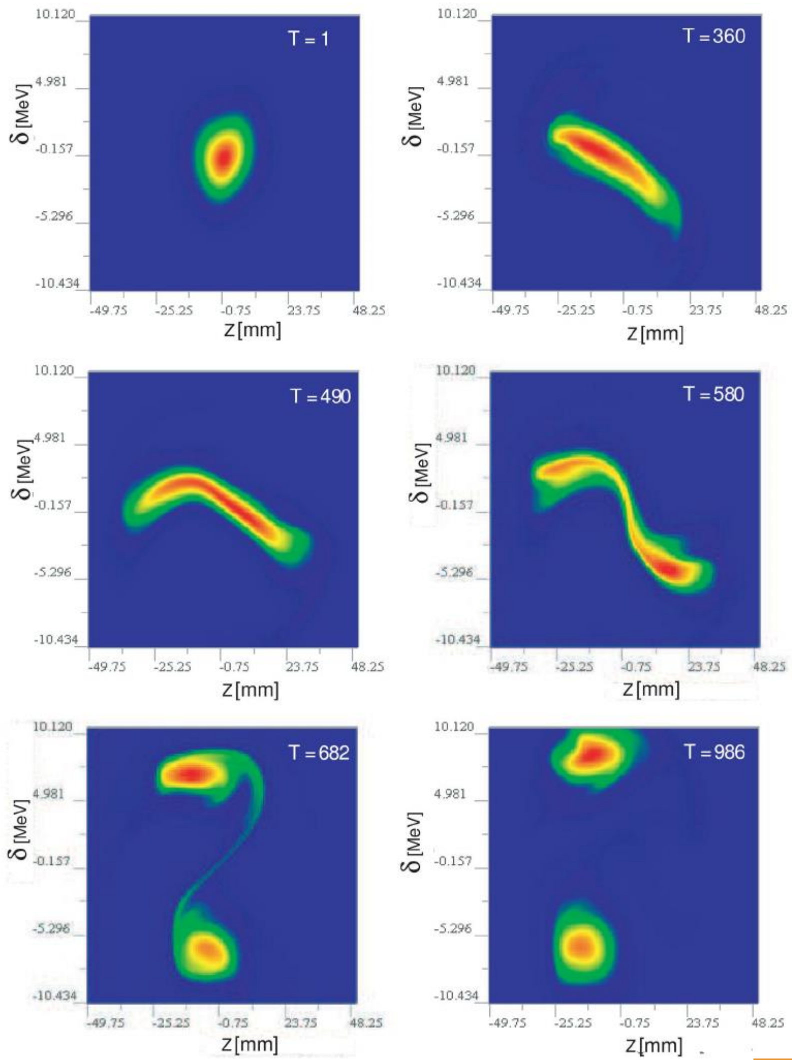
- $Z_{||}$: Longitudinal impedance.
- n: Harmonic number.
- α: Momentum compaction factor.
- E: Beam energy.
- δ: Relative energy spread.
- I_0: Beam current.

This formula is a foundational tool in impedance budgeting.

Head-Tail Instability

Head-tail instability is a transverse collective effect where the tail of a bunch, driven by wake fields left by its head, oscillates out of phase, leading to growing betatron oscillations above a certain current threshold.

- This is a single-bunch effect driven by short-range transverse wake fields.
- **Primary Sources:** Head-tail instability is a transverse beam instability driven by wakefields. This interaction perturbs the beam, which can lead to a variation in the betatron tune with current. This type of instability is more easily met in trailing beams than in test beams.
- **Design Challenges:** The pursuit of ultra-low emittance in modern synchrotrons has led to designs with smaller vacuum chamber apertures, which increases the machine's coupling impedance and makes the beam more sensitive to instabilities. In low-emittance machines, instabilities that "explode the emittance" are a serious concern.



E. Karantzoulis and M. Lonza, "*Transverse Head-tail Modes Elimination with Negative Chromaticity and the Transverse Multi-bunch Feedback System at ELETTRA*", in Proc. EPAC'06, Edinburgh, UK, Jun. 2006, paper THPCH045, pp. 2886-2888.

Head-Tail Instability

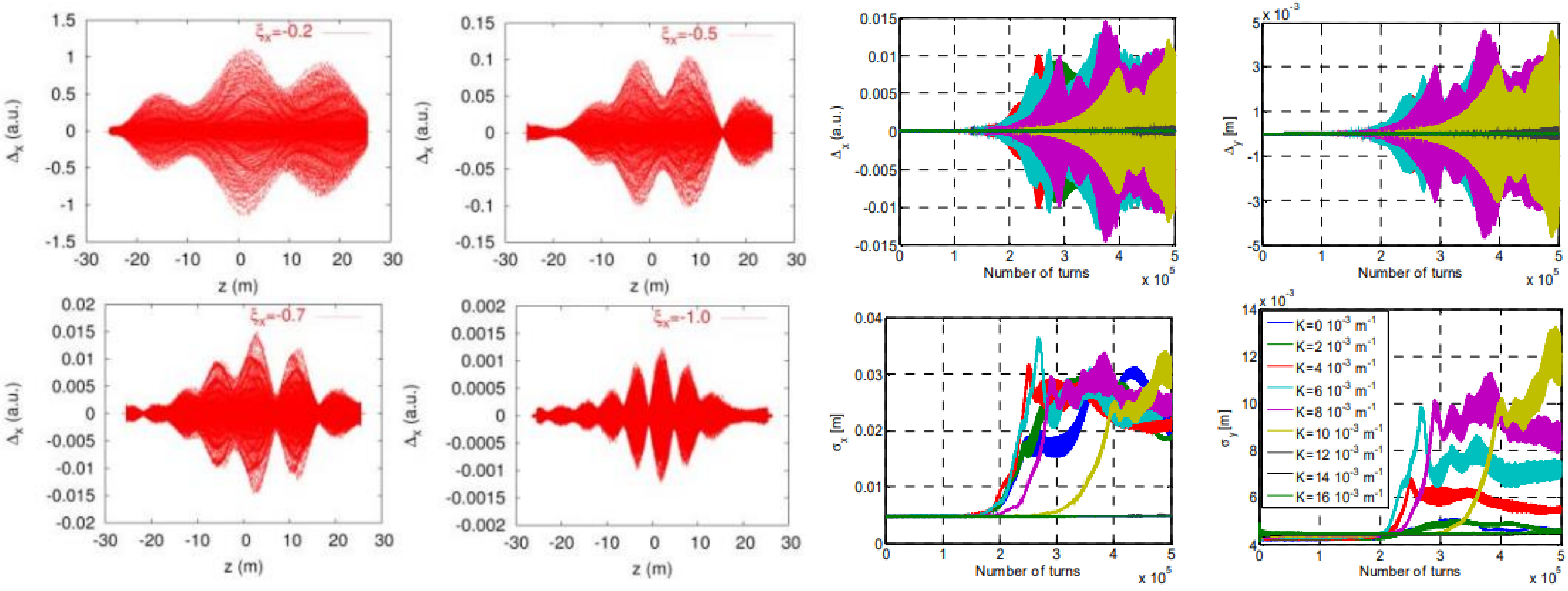
- **Mode coupling** between transverse betatron motion and synchrotron oscillations.
- Strong dependence on **chromaticity** ξ, synchrotron tune, and transverse impedance.
- A simplified threshold criterion for the onset of head–tail instability is:
- ξ: Chromaticity;
- I_b: Bunch current;
- ν_s: Synchrotron tune;
- *R*: Average ring radius.

$$|\xi| < \left|\frac{Z_\perp}{n}\right| \cdot \frac{I_b R}{E_0 I \nu_s},$$

- **Mitigation: S**everal mechanisms are used to suppress head-tail instabilities. Landau damping, which is a mechanism of natural decoherence, has been shown to be effective against both single-bunch and coupled-bunch instabilities. The instability threshold can also be controlled by adjusting the machine's chromaticity. Additionally, feedback systems, such as bunch-by-bunch feedback, can be used to control the instability in machines operating with low chromaticity. Space charge effects can also play a role in suppressing head-tail instabilities by modifying the frequencies of the bunch modes.

Heifets, S. & Novokhatski, A.. (2006). Coherent beam stability in the low momentum compaction lattice. Physical Review Special Topics - accelerators and Beams - PHYS REV SPEC TOP-AC. 9. 10.1103/PhysRevSTAB.9.044402.

Visualizing Head-Tail Instability



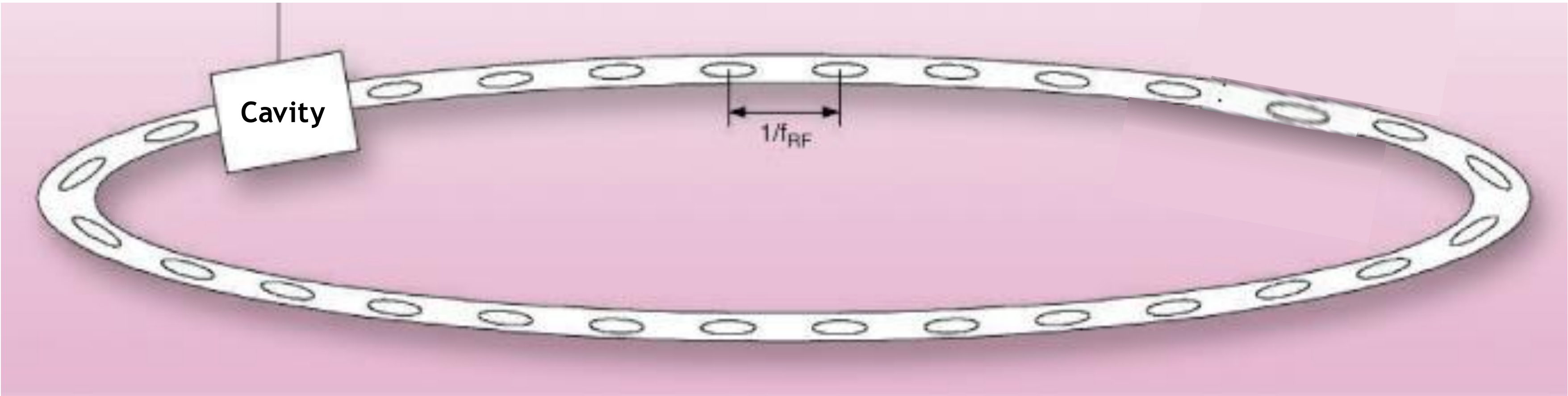
- The leading slice oscillating in one direction, causing a wake field that kicks the next slice in the opposite direction.
- Over time, the oscillations would grow in amplitude, showing the instability.

E. Metral, G. Rumolo, B. Salvant, and R. R. Steerenberg, “Simulation Study of the Horizontal Head-Tail Instability Observed at Injection of the CERN Proton Synchrotron”, in Proc. PAC'07, Albuquerque, NM, USA, Jun. 2007, paper FRPMN074, pp. 4210-4212.

Coupled-Bunch Instabilities (CBIs)

Coupled-bunch instabilities are collective oscillations where wake fields left by one bunch interact with following bunches, causing their motions to become synchronized and unstable across multiple bunches in the ring.

- This is a multi-bunch effect caused by long-range wake fields from narrowband sources.
- **Primary Sources:** This type of instability is caused by wakefields from a leading electron bunch that last long enough to affect subsequent bunches in the accelerator ring. A primary source is the narrow-band impedance of radio-frequency (RF) cavities, particularly from their higher-order modes (HOMs), which can drive longitudinal coupled-bunch instabilities (LCBI). Resistive wall impedance from the vacuum chamber can also drive these instabilities.



CBI Growth Rate

- **Design Challenges:** The high beam currents used in modern accelerators lead to a strong coupling of bunch motion with the cavity impedance, which can overcome natural damping and cause instabilities. In fourth-generation light sources like the ESRF-EBS, a lower instability threshold for LCBI required a new approach to cavity design. In certain cases, such as in BESSY II, the influence of collective effects is significantly lower in multi-bunch mode than in single-bunch mode.
- The growth rate for a given mode is directly proportional to the resistive part of the impedance at the mode's frequency:

$$\frac{1}{\tau_\mu} = \frac{eI_0}{2E_0T_0} \cdot \mathrm{Re}[Z_\perp(\omega_\mu)]$$

- $1/\tau_\mu$: The growth rate of the instability.
- $\mathrm{Re}[Z_\perp(\omega_\mu)]$: The resistive (energy-dissipating) part of the impedance at the mode's frequency.
- **Mitigation:** Mitigation strategies include both passive design and active systems. Passive solutions include designing strongly HOM-damped RF cavities to push the instability threshold above the nominal beam current. At the MAX IV facility, a low RF frequency (100 MHz) is used to produce long bunches, which are further lengthened by passively operated third-harmonic Landau cavities to alleviate these instabilities. Active countermeasures include fast RF feedback systems that can compensate for beam-induced fields, thereby significantly reducing the effective impedance of the RF cavities. Landau damping is also an effective natural mechanism for suppressing both single-bunch and coupled-bunch instabilities.

Transverse Mode Coupling Instability (TMCI)

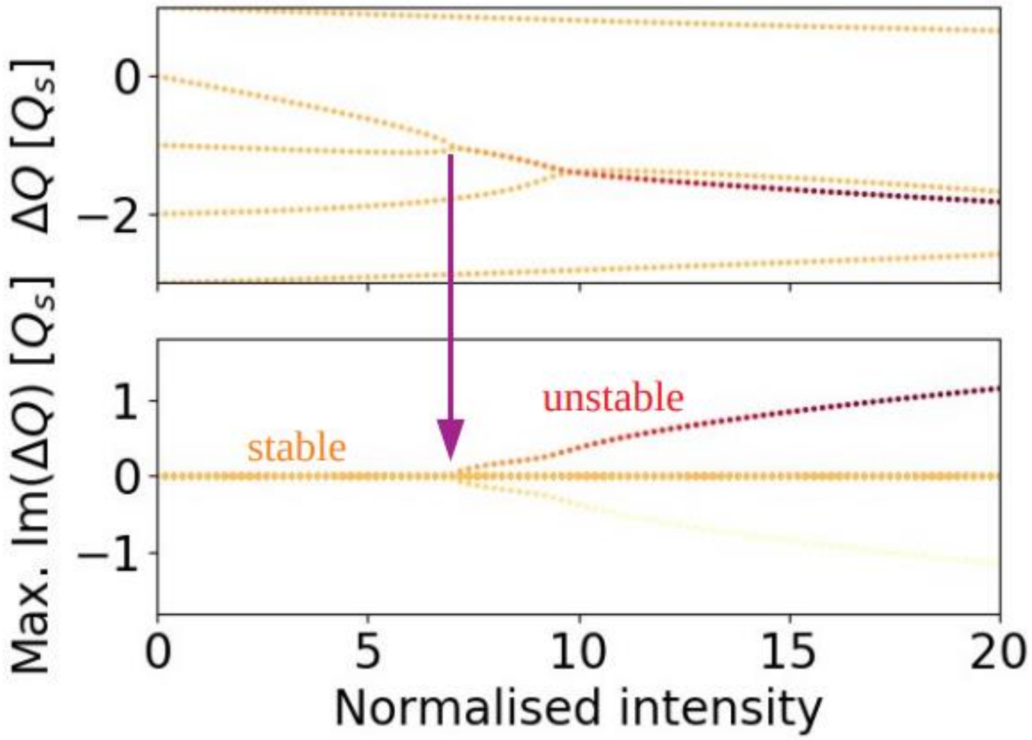
Transverse Mode Coupling Instability (TMCI) occurs when increasing beam current causes different betatron oscillation modes to couple, leading to a rapid growth of transverse oscillations and beam loss above a sharp threshold.

- TMCI is a single-bunch instability driven by **broadband impedance.**
- **Primary Sources:** TMCI is a collective, single-bunch instability that limits bunch intensity. It is driven by wakefields that cause a coupling between different modes of coherent oscillation. The instability threshold is reached when the frequency of one mode merges with that of an adjacent mode, causing the beam's transverse oscillations to grow exponentially.
- **Design Challenges:** The low-emittance, high-current designs of modern synchrotrons inherently increase a beam's susceptibility to instabilities. TMCI, specifically, can cause a rapid increase in beam size and emittance, leading to a significant degradation of performance above a certain threshold. Historically, theoretical models have produced conflicting results on the behavior of TMCI at high space charge, complicating design efforts.

M. Venturini, Harmonic cavities and the transverse mode-coupling instability driven by a resistive wall. *Physical Review Accelerators and Beams*, 21(2), 024402.

TMCI

- **Mitigation:** The TMCI threshold can be controlled by adjusting the machine's chromaticity. Space charge (SC) can also be a stabilizing force by altering the bunch's coherent spectra. A new theoretical framework shows that for a common class of wakes, the TMCI threshold increases indefinitely with the space charge tune shift, providing a powerful stabilizing effect that can more than double the threshold current. Additionally, passive harmonic cavities can be used for bunch lengthening, which helps suppress instabilities like TMCI, and Landau damping is a natural mechanism of decoherence that provides stability.



https://conference.sns.gov/event/335/contributions/501/attachments/866/8131/2022-10-24_Knoxville.pdf

M. Venturini, Harmonic cavities and the transverse mode-coupling instability driven by a resistive wall. *Physical Review Accelerators and Beams*, 21(2), 024402.

Table 3: Overview of Instability Mechanisms

Instability Type	Dominant Impedance	Key Parameters	Typical Mitigation
Head-Tail (single-bunch)	Transverse broadband	Chromaticity, wake slope	Positive chromaticity (ξ), feedback
TMCI (single-bunch)	Transverse broadband	Wake amplitude, bunch length	Bunch lengthening (e.g., harmonic cavities), feedback
Microwave (SB)	Longitudinal broadband	$Z_{\|\|}/n$, δ	Harmonic cavities, damping
Coupled-Bunch (multi-bunch)	Longitudinal or transverse narrowband	HOM frequency, Q, mode number	HOM dampers, detuning, active feedback

Mitigation & Tools

緩和とツール

Instability Mitigation

Strategies for a Stable Beam

- **Passive Solutions:**
  - HOM dampers and frequency detuning.
  - Low-impedance vacuum chamber designs (e.g., "ultra-smooth" chambers).
  - RF shielding for bellows.
- **Active Solutions:**
  - Bunch-by-bunch feedback systems that detect and correct oscillations in real-time.
  - Modulating chromaticity to induce Landau damping.

1. **C.M. Bhat and N. Eddy**, *"Coupled Bunch Instabilities Growth in the Fermilab Booster During Acceleration Cycle"* in Proc. 64th Adv. Beam Dyn. Workshop High-Intensity High-Brightness Hadron Beams (HB'21), Batavia, IL, USA, Oct. 2021, pp.~140-145.
2. **I. Zagorodnov**, *Wakefield code ECHO: Algorithms and applications*. Presentation at the Second Topical Workshop on Instabilities, Impedance and Collective Effects (TWIICE 2), Abingdon, Oxfordshire, UK, February 8, 2016.

The Shift from Passive to Active Control

- **Past:** Relying on passive, low-impedance designs (e.g., smooth chambers).
- **Present:** A sophisticated, multi-pronged approach that combines passive design with powerful active systems.
- The advent of modern digital feedback systems has enabled machine designers to tolerate some level of unavoidable impedance by actively damping the resulting instabilities in real-time.

Natural and Active Damping

Fighting Back Against Growth

- **Radiation Damping:** An inherent stabilizing force in storage rings where particles lose energy via synchrotron radiation.
- **Landau Damping:** Arises from a spread in betatron or synchrotron frequencies, providing phase mixing that dampens coherent motion.
- **Feedback Systems:** Provide active damping by detecting and correcting oscillations in real-time. This is essential for stabilizing the highest-current beams.

How Engineers Study Impedance

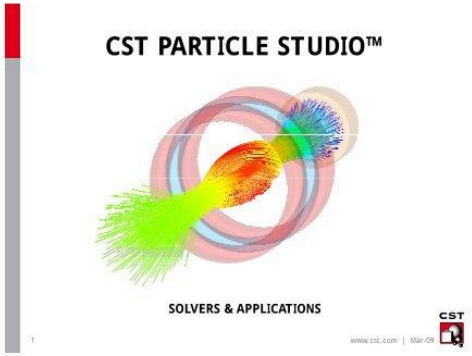
Analytical and Numerical Tools

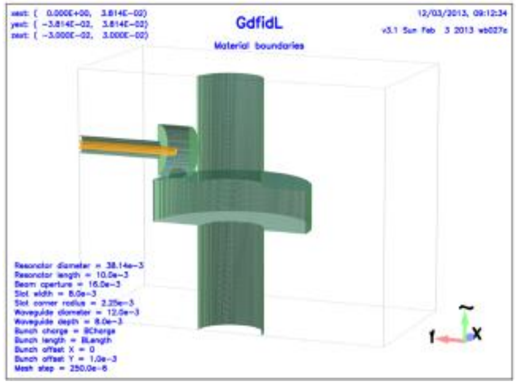
- For simple geometries, analytical formulas can provide a good estimate.
- For the complex shapes of modern accelerators, numerical electromagnetic solvers are essential for accurate evaluation.
- These tools are used to build detailed impedance databases for modern facilities.

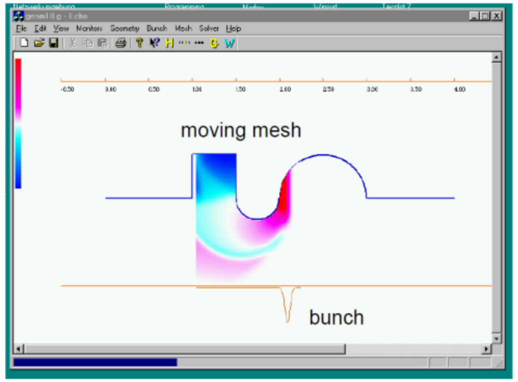
How Engineers Study Impedance

Time-Domain Solvers

- **What they do:** Simulate the passage of a charged bunch through a structure to compute the wake potential as a function of time.
- **When to use:** Well-suited for calculating broadband impedance and short-range wake fields.
- **Examples:** CST Particle Studio (Wakefield), GdfidL, ECHO.







ECHO

Electromagnetic
Code for
Handling
Of
Harmful
Collective
Effects

How Engineers Study Impedance

Frequency-Domain Solvers

- **What they do:** Solve Maxwell's equations at fixed frequencies to reveal the resonant properties of a structure.
- **When to use:** Ideal for analyzing high-Q resonant structures and identifying HOMs.
- **Examples:** CST Particle Studio (Frequency Domain), ANSYS HFSS, Superfish.

1. **L. Emery**, “*Coupled-Bunch Instability Study of Multi-cell Deflecting Mode Cavities for the Advanced Photon Source*”, in Proc. PAC'07, Albuquerque, NM, USA, Jun. 2007, paper FRPMN108, pp. 4348-4350.
2. **Migliorati, M., Biancacci, N., Masullo, M. R., Palumbo, L., & Vaccaro, V. G.** (2018). *Space charge impedance and electromagnetic fields in elliptical vacuum chambers*. Physical Review Accelerators and Beams, 21 (12), 124201.
3. **X. Buffat and H. Bartosik**, “*Transverse Mode Coupling Instability with Space Charge at the CERN SPS,*” presented at the 5th ICFA Mini-Workshop on Space Charge, Knoxville, Tennessee, USA, Oct. 24–26, 2022.

Table 1: Numerical Solvers

Code	Domain	Description
CST Particle Studio (Wakefield)	Time	Commercial 3D electromagnetic PIC solver for wake potential calculation, used for broadband impedance.
GdfidL	Time	Finite-Difference Time-Domain (FDTD) solver tailored for wake field and impedance problems, particularly for complex geometries.
ECHO / ECHO2D	Time	Time-domain code for wake field calculation in axisymmetric structures.
CST Particle Studio (Frequency Domain)	Frequency	FEM-based solver, efficient for eigenmode and S-parameter analysis of resonant structures.
HFSS (Ansys)	Frequency	Finite Element Method (FEM) solver for full 3D structures, widely used for HOM analysis and S-parameter calculations.
Superfish	Frequency	2D eigenmode solver, best for axisymmetric RF cavities.

Conclusion

結論

Synthesis: The Big Picture

1. The Shift from Passive to Active Control

- The field has evolved from relying on passive, low-impedance designs to a multi-pronged approach combining passive design with powerful active feedback systems.
- Modern digital feedback systems enable designers to tolerate some level of unavoidable impedance.

2. The Interconnected Nature of Accelerator Physics

- A seemingly straightforward action in one plane of motion can have unintended, and at times contradictory, effects in another.
- Example: A solution for one problem (bunch lengthening to address microwave instability) can reduce the TMCI threshold and worsen another problem.
- This complexity necessitates an integrated design approach.

Summary & Conclusion

- Impedance is a central, defining challenge in modern synchrotron light sources.
- Its effects on the beam—including energy loss, tune shifts, and instabilities—are not just theoretical but have direct, measurable consequences on performance.
- A deep understanding requires a combination of analytical theory, advanced numerical simulation, and beam-based diagnostics.
- Effective impedance control is a cornerstone of ensuring stable, high-brightness operation for future light sources.

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1. **W.K.H. Panofsky and W.A. Wenzel**, "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields," Review of Scientific Instruments, 1956.
2. **V.G. Vaccaro**, "Longitudinal Instability of a Coasting Beam," CERN ISR-RF/66-35, 1966.
3. **E. Keil and W. Schnell**, "Concerning longitudinal stability in the ISR," CERN-ISR-TH-RF-69-48, 1969.
4. **E. Keil, and W. Schnell**, "Collective Instabilities and High Frequency Resistive Wall Effects in Proton Storage Rings," CERN Report, CERN 69-15, 1969.
5. **D. Boussard**, "Observation of microwave longitudinal instability in the CPS proton storage ring," IEEE Transactions on Nuclear Science, 1983.
6. **K.L.F. Bane, & B.W. Zotter**, "Impedances of small discontinuities for round and flat chambers," Particle Accelerators, 1990.
7. **A.W. Chao**, Physics of Collective Beam Instabilities in High Energy Accelerators, 1993.
8. **R. L. Warnock**, "Coherent synchrotron radiation and bunch lengthening in electron storage rings," Nuclear Instruments and Methods in Physics Research Section A, 1994.
9. **S. Khan**, "Beam impedance study for the BESSY II storage ring," Proceedings of the 1997 Particle Accelerator Conference (PAC'97), 1998.
10. **B. Zotter, & S. Kheifets**, Impedances and Wakes in High-Energy Particle Accelerators, World Scientific, 1998.Another essential textbook that serves as a standard reference for both theorists and experimentalists working in the field.